MATH 520 (Section 001)
Prof. Meade

Exam 1
8 February 2013

University of South Carolina
Spring 2013

Name: Key

Instructions:

1. There are a total of 5 problems on 3 pages. Check that your copy of the exam has all of the problems.
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.
5. Your answers must be clearly labeled and written legibly on additional sheets of paper (that I will provide). Be sure each sheet contains your name and the work for each question is clearly labeled.
6. Check your work. If I see *clear evidence* that you checked your answer (when possible) <u>and</u> you *clearly indicate* that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

Problem	Points	Score
1	15	
2	20	
3	32	
4	21	
5	12	
Total	100	

Good Luck!

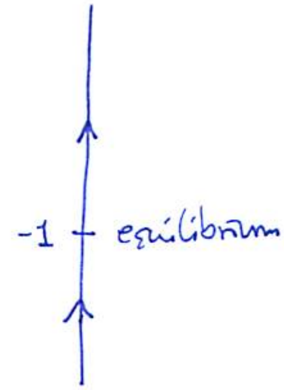
Exam 1 - Solutions

#1. $y' = (y+1)^2 e^{y-2}$

equilib. sol'n: $y' = 0$

$(y+1)^2 e^{y-2} = 0 \Rightarrow y = -1.$

$\left\{ \begin{array}{l} (y+1)^2 > 0 \text{ unless } y = -1 \\ e^{y-2} > 0 \text{ for all } y. \end{array} \right\}$



$$\lim_{t\to\infty} y(t) = \begin{cases} +\infty & \text{if } y(0) > -1 \\ -1 & \text{if } y(0) \le -1. \end{cases}$$

#2. see exam pages.

#3. (a) $t^2 y' + (t+2) y = e^t$

std form: $y' + \frac{t+2}{t^2} y = \frac{e^t}{t^2}$

integrating factor:

$\mu' = \frac{t+2}{t}\mu$

$\frac{d\mu}{\mu} = \left(\frac{t+2}{t}\right) dt = \left(1 + \frac{2}{t}\right) dt$

$\ln \mu = t + 2\ln t$

$\mu = e^{t+2\ln t} = e^t e^{2\ln t}$

$= e^t e^{\ln t^2} = t^2 e^t.$

$t^2 e^t y' + t(t+2) e^t y = e^{2t}$

$\int \frac{d}{dt}\left(t^2 e^t y\right) = \int e^{2t}$

$t^2 e^t y = \frac{1}{2} e^{2t} + C$

$y = \frac{e^t}{2t^2} + \frac{C}{t^2 e^t}$

(b) $\frac{dy}{dt} = e^{3t+2y} = e^{3t} e^{2y}$

$\frac{dy}{e^{2y}} = e^{3t} dt$

$-\frac{1}{2} e^{-2y} = \frac{1}{3} e^{3t} + C$

$e^{-2y} = -\frac{2}{3} e^{3t} + C$

apply the IC: $y(0) = 0$

$e^{-2\cdot 0} = -\frac{2}{3} e^{3\cdot 0} + C$

$1 = -\frac{2}{3} + C$ so $C = \frac{5}{3}.$

$e^{-2y} = -\frac{2}{3} e^{3t} + \frac{5}{3}$

$-2y = \ln\left(\frac{5}{3} - \frac{2}{3} e^{3t}\right)$

$y = -\frac{1}{2} \ln\left(\frac{5}{3} - \frac{2}{3} e^{3t}\right)$

provided $\frac{5}{3} - \frac{2}{3} e^{3t} > 0$

$\frac{5}{3} > \frac{2}{3} e^{3t}$

$\frac{5}{2} > e^{3t}$

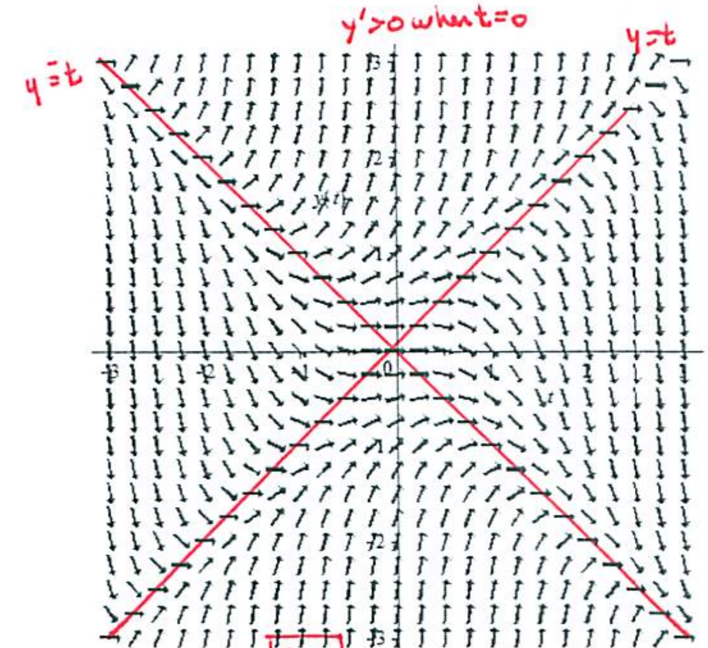
$3t < \ln(5/2) \quad \therefore \quad t < \frac{1}{3} \ln\left(\frac{5}{3}\right)$

1. (15 points) Draw the direction line for the differential equation $y' = (y+1)^2 e^{y-2}$. Use this information to determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe this dependency.

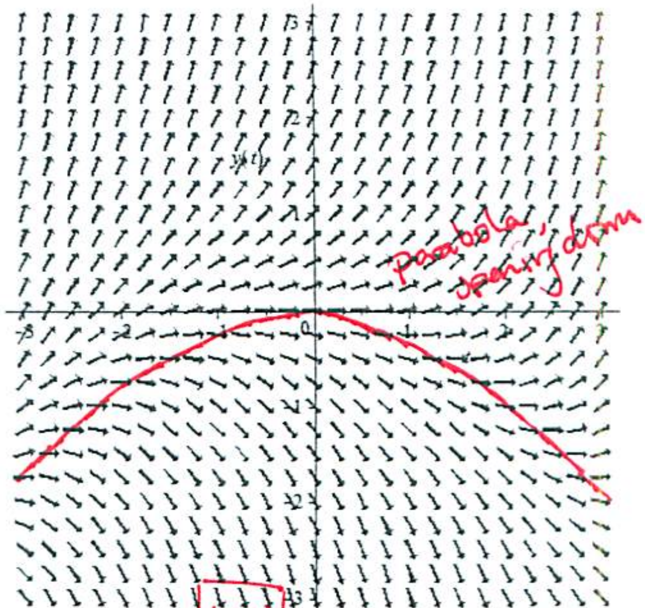
2. (20 points) Consider the following list of differential equations.

$y=-t$ (i) $y' = t + y$	$y=t$ (ii) $y' = t - y$	$y=-0.2t^2$ (iii) $y' = 0.2t^2 + y$	$y=0$ or $y=2$ (iv) $y' = y(1 - y/2)$
(v) $y' = y^2 - t^2$ $y=\pm t$	(vi) $y' = t^2 - y^2$ $y=\pm t$	(vii) $y' = 1 - ty$ $t = 1/t$	(viii) $y' = \sin(t)\cos(t)$ $t = n\frac{\pi}{2}$ (n an integer)

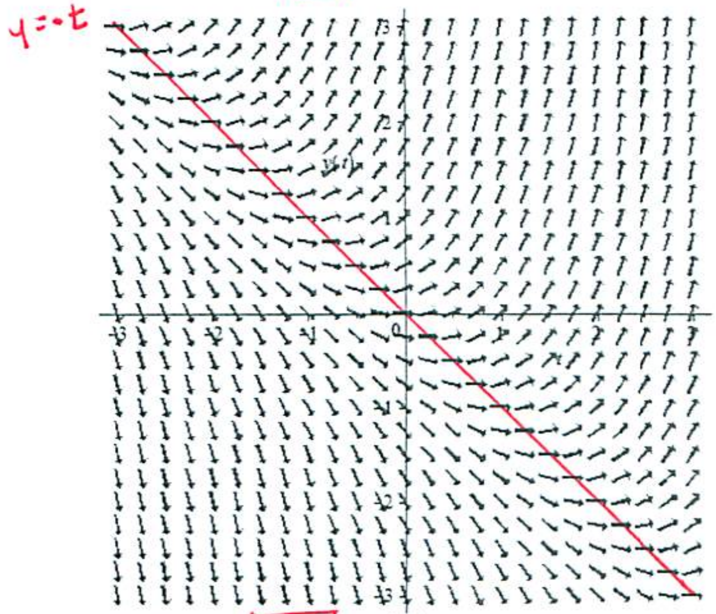
Identify the differential equation that corresponds to each direction field.



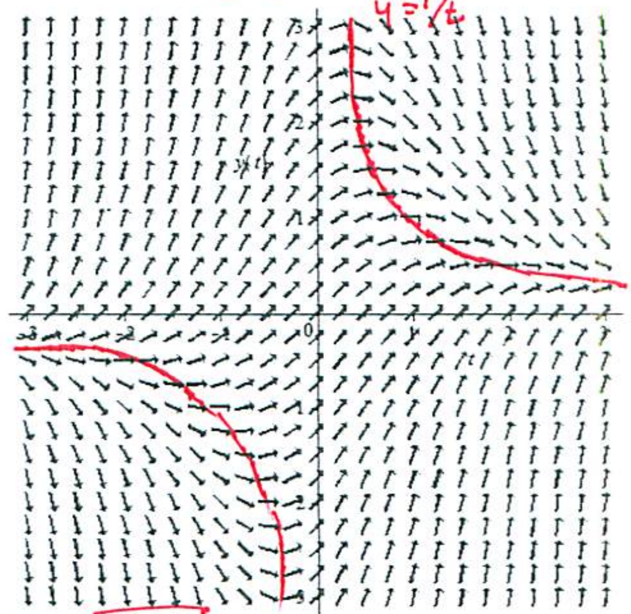
(v) (A)



(iii) (B)



(i) (C)



(vii) (D)

4. $\frac{dy}{dt} + \frac{1}{t^2-6t+8}y = \sqrt{t+1}$ linear DE

$p(t) = \frac{1}{t^2-6t+8} = \frac{1}{(t-2)(t-4)}$ continuous for $t \neq 2$ & $t \neq 4$.

$g(t) = \sqrt{t+1}$ continuous for $t > -1$.

a) $y(0) = 3$

largest interval where p & g are continuous that contains $t_0 = 0$ is $\underline{(-1, 2)}$

b) $y(2) = 1$

~~largest interval~~

because $p(t)$ is not continuous at $t_0 = 2$,
<u>the Theorem does not say anything about this case.</u>

d) $y(5) = -3$

the largest interval containing $t_0 = 5$ where p & g are continuous is $\underline{(4, \infty)}$

5. $\underbrace{6xy^3 + \cos y}_{M} + (\underbrace{3bx^2y^2 - x\sin y}_{N})\frac{dy}{dx} = 0$

To be exact requires $M_y = N_x$: $M_y = 18xy^2 - \sin y$

$N_x = 6bxy^2 - \sin y = M_y$ when $\underline{b = 3}$

$(\underbrace{6xy^3 + \cos y}_{M}) + (\underbrace{9x^2y^2 - x\sin y}_{N})\frac{dy}{dx} = 0$

A solution can be written in the form $\psi(x,y) = C$
where $\psi_x = M$ and $\psi_y = N$.

$\psi_x = 6xy^3 + \cos y \implies \psi = 3x^2y^3 + x\cos y + h(y)$

$\psi_y = 9x^2y^2 - x\sin y + h'(y) = 9x^2y^2 - x\sin y$

when $h'(y) = 0$

so $h(y) = 0$. $\therefore \underline{3x^2y^3 + x\cos y = C}$