MATH 520 (Section 001)
Prof. Meade

University of South Carolina
Fall 2011

Exam 1
September 20, 2011

Name: ____________________
SS # (last 4 digits): __________

Instructions:

1. There are a total of 5 problems on 3 pages. Check that your copy of the exam has all of the problems.
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.
5. Your answers must be clearly labeled and written legibly on additional sheets of paper (that I will provide). Be sure each sheet contains your name and the work for each question is clearly labeled.
6. Check your work. If I see *clear evidence* that you checked your answer (when possible) <u>and</u> you *clearly indicate* that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

| Problem | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 20 | |
| 3 | 36 | |
| 4 | 20 | |
| 5 | 14 | |
| Total | 100 | |

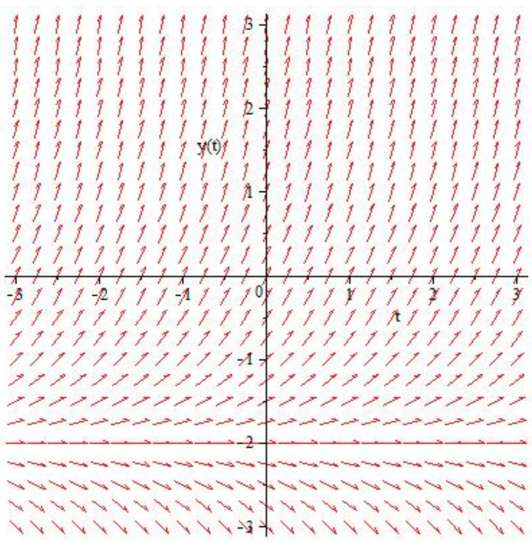
Good Luck!

1. (10 points) Draw the direction line for the differential equation $y' = (3-y)^2(y+1)$. Use this information to determine the behavior of $y$ as $t \to \infty$. If this behavior depends on the initial value of $y$ at $t = 0$, describe this dependency.

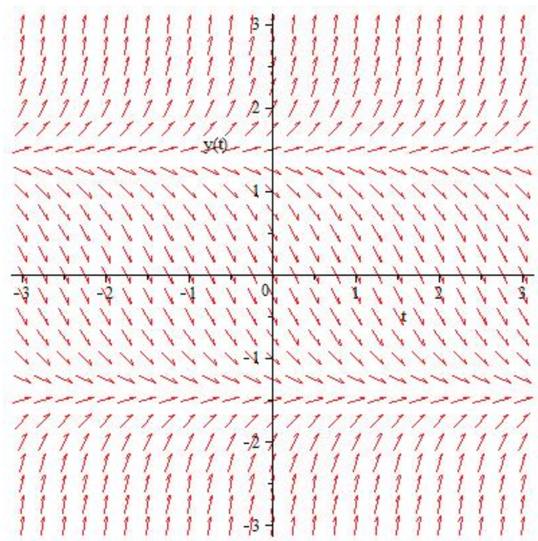
2. (20 points) Consider the following list of differential equations.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (i) | $y' = 2 - y^2$ | (ii) | $y' = 2t - 2$ | (iii) | $y' = 2 - y$ | (iv) | $y' = 2t - 1$ |
| (v) | $y' = y^2 - t^2$ | (vi) | $y' = t^2 - y^2$ | (vii) | $y' = 2 + y$ | (viii) | $y' = y^2 - 2$ |

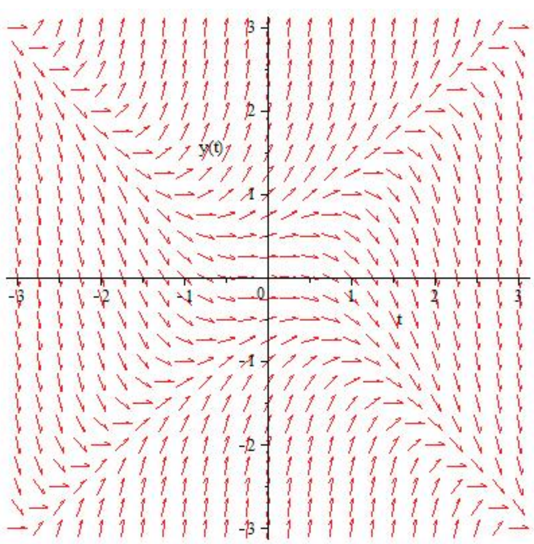
Identify the differential equation that corresponds to each direction field.



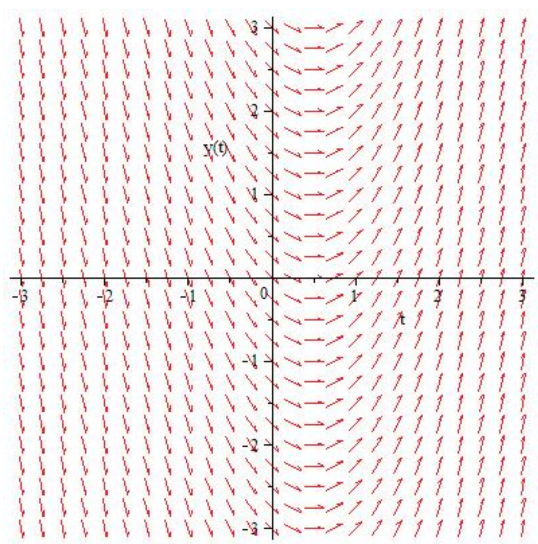

(A)




(B)




(C)




(D)

3. (36 points) Find the general solution of each differential equation. If an initial condition is given, find the solution to the initial value problem. If possible, solve for $y$.

   (a) $ty' - y = t^2e^{-t}, \quad t > 0$

   (b) $e^x \sin(y) + 3y + (3x + y^2 + e^x \cos(y))y' = 0$

   (c) $y' = xy^2(1 - x^2)^{-1/2}$

4. (20 points)

   (a) Determine an interval in which the solution of the given initial value problem is certain to exist:

$$(\cos(t))y' + (\sin(t))y = \cos(t)\sin(t), \qquad y(\pi) = \frac{\sqrt{3}}{2}.$$

   (b) For what initial conditions, $y(t_0) = y_0$, are the hypotheses of the Existence and Uniqueness Theorem satisfied for the differential equation.

$$y' = \frac{\ln|ty|}{1 - t^2 - y^2}.$$

5. (14 points) Show that the differential equation is not exact, but becomes exact when multiplied by the given integrating factor. Then solve the equation.

$$(x\sec(y) + 3x^3\tan(y)) + (xy^2\sec(y) + x^4)\frac{dy}{dx} = 0, \qquad \mu = \frac{\cos(y)}{x}.$$