[o my late parents, Anatole A. Solow and Ruth Solow
and to my wife of many years, Audrey

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Preface to the Student

After finishing my undergraduate degree, I began to wonder why learning theoretical mathematics had been so difficult. As I progressed through my graduate work, I realized that mathematics possessed many of the aspects of a game—a game in which the rules had been partially concealed. Imagine trying to play chess before you know how all of the pieces move! It is no wonder that so many students have had trouble with abstract mathematics.

This book describes some of the rules by which the game of theoretical mathematics is played. It has been my experience that virtually anyone who is motivated and who has a knowledge of high school mathematics can learn these rules. Doing so greatly reduces the time (and frustration) involved in learning abstract mathematics. I hope this book serves that purpose for you.

To play chess, you must first learn how the individual pieces move. Only after these rules have entered your subconscious can your mind turn its full attention to the more creative issues of strategy, tactics, and the like. So it appears to be with mathematics. Hard work is required in the beginning to learn the fundamental rules presented in this book. To that end, in addition to reading the material in this book and working through as many exercises as possible (as there is no substitute for practice), you can also access a collection of videotaped lectures, one for each of the first 15 chapters of the book, on the web at www.wiley.com/college/solow/.

Your goal should be to absorb the material in this book so that it becomes second nature to you. Then you will find that your mind can focus on the creative aspects of mathematics. These rules are no substitute for creativand this book is not meant to teach creativity. However, I do believe ; the ideas presented here provide you with the tools needed to express r creativity. Equally important is the fact that these tools enable you ınderstand and appreciate the creativity of others. To that end, much ›hasis is placed on teaching you how to read "condensed" proofs as they typically presented in textbooks, journal articles, and other mathematical ·ature. Knowing how to read and understand such proofs enables you to milate the material in any advanced mathematics course for which you e the appropriate prerequisite background. In fact, knowing how to read understand condensed proofs gives you the ability to learn virtually any ;hematical subject on your own, with enough time and effort.

´ou are about to learn a key part of the mathematical thought process. As study the material and solve problems, be conscious of your own thought cesses. Ask questions and seek answers. Remember, the only unintelligent stion is the one that goes unasked.

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Preface to the Instructor

The Objective of This Book

The inability to communicate proofs in an understandable manner has plagued students and teachers in all branches of mathematics. The result has been frustrated students, frustrated teachers, and, oftentimes, a watered-down course to enable the students to follow at least some of the material or a test that protects students from the consequences of this deficiency in their mathematical understanding.

One might conjecture that most students simply cannot understand abstract mathematics, but my experience indicates otherwise. What seems to have been lacking is a proper method for explaining theoretical mathematics. In this book I have developed a method for communicating proofs—a common language that professors can teach and students can understand. In essence, this book categorizes, identifies, and explains (at the student's level) the various techniques that are used repeatedly in virtually all proofs.

Once the students understand the techniques, it is then possible to explain any proof as a sequence of applications of these techniques. In fact, it is advisable to do so because the process reinforces what the students have learned in the book.

Explaining a proof in terms of its component techniques is not difficult, as is illustrated in the examples of this book. Before each "condensed" proof is an analysis explaining the methodology, thought processes, and techniques that are used. Teaching proofs in this manner requires nothing more than eding each step of the proof with an indication of which technique is about e used and why. When discussing a proof in class, I actively involve the ents by soliciting their help in choosing the techniques and designing the of. I have been pleasantly surprised by the quality of their comments and stions.

n addition to the collection of proof techniques in Part I, I have identi- in Part II a number of other *mathematical thinking processes* that are l in virtually all college-level math courses. These thinking processes were introduced in my book *The Keys to Advanced Mathematics* in 1995 and ude:

- Generalization and unification.
- Identifying similarities and differences.
- Creating a visual image for a mathematical concept and, vice versa, converting a visual image of a mathematical concept to a written symbolic form.
- Creating definitions.
- Learning to use abstraction.
- Developing and working with axiomatic systems.

viding the student with these thinking processes appears to facilitate the dent's ability to learn subsequent mathematical material.

t has been my experience that once students become comfortable with proof techniques and these other thinking processes, their minds tend to lress the more important issues of mathematics, such as why a proof is e in a particular way and why the piece of mathematics is important in first place. This book is not meant to teach creativity, but I do believe t learning the techniques presented here frees the student's mind to focus the creative aspects. I have also found that, by using this approach, it is sible to teach subsequent mathematical material at a more sophisticated el without losing the students.

In any event, the message is clear. I am suggesting that there are many efits to be gained by teaching mathematical thought processes in addition mathematical material. This book is designed to be a major step in the ht direction by making abstract mathematics understandable and enjoyable the students and by providing you with a method for communicating with m.

at's New in the Sixth Edition

ere are two primary changes in the sixth edition of this book. The first the inclusion of a new Part II that contains a description of the afore-

mentioned mathematical thinking processes. As with the proof techniques, a name is given to each of the thinking processes which are then described at the student's level with easy-to-understand examples. These examples, together with numerous exercises, are designed to give the student practice in understanding and using these thinking processes so that the student will be aware of these techniques when they arise in their subsequent math courses.

Although these changes seem to make it even easier for students to understand proofs and advanced mathematical subject matter, I have still found no substitute for actively teaching the material in class instead of having the students read the material on their own. This active interaction has proved eminently beneficial to both student and teacher, in my case. However, it often happens that there is not enough time in a given course to teach the proof techniques as well as other requisite mathematical subject matter. To address this challenge, I have included with the sixth edition, videotaped lectures for each proof technique that students can watch at their own pace on the web at www.wiley.com/college/solow/. I hope these lectures aid the students in learning how to read and do proofs.

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