Definitions

Chapter 1

- **Proof Technique (Pg.1):** Any method for proving that the statement " A implies B " is true.
- **Proof (Pg.2):** A convincing argument expressed in the language of mathematics that a statement is true.
- **Statement (Pg.2):** A sentence that is either true or false.
- **Conditional Statement/Implication (Pg3.):** A statement of the form, "If A is true, then B is true," where A and B are given statements.
- **Hypothesis(Pg.3):** The statement A in the implication " A implies B ." When proving " A implies B ," you can assume that the hypothesis A is true.
- **Conclusion(Pg.3):** The statement B in the implication " A implies B ." When proving " A implies B ," your job is to show that the conclusion B is true.
- **Truth Table (Pg.4):** A table that lists the truth of a complex statement (such as " A implies B ") for all possible combinations of truth values of the individual statements (in this case, A and B).

Chapter 2

- **Forward-Backward Method (Pg.9):** The technique for proving that " A implies B " in which you assume that A is true and try to show that B is true. To do so, you apply the forward process to A and the backward process to B.
- **Backward Process (Pg.9):** The process of deriving from a statement, B, a new statement, B1, with the property that, if B1 is true, then so is B. You do this by asking and answering a key question.
- **Forward Process (Pg.9):** The process of deriving from a statement, A , a new statement, A 1, with the property that A 1 is true because A is true.
- **Key Question (Pg.10):** The specific question obtained by asking how you can show that a given statement B is true.
- **Abstract Answer (Pg.10):** An answer that contains no symbols from the specific problem.
- **Analysis of Proof(Pg.15):** The techniques, methodology and reasoning involved in doing the proof.

Chapter 3

- **Definition (Pg.25):** An agreement, by all parties concerned, as to the meaning of a particular term.
- **Divides (Pg.26):** An integer n divides an integer m (written $n \mid m$) if $m = kn$, for some integer k.
- **Prime Integer (Pg.26):** An integer $p > 1$ is prime if the only positive integers that divide p are 1 and p.

- **Equal(Pg.26):** Two ordered pairs of real numbers (x_1 ,x_2) and (y_1 ,y_2) are equal if $x_1 = y_2$ and $x_2 = y_2$.
- **Even Integer (Pg.26):** An integer n whose remainder on dividing by 2 is 0. Equivalently, an integer n for which there is an integer k such that n = 2k.
- **Odd Integer (Pg.26):** An integer n for which there is an integer k such that n = 2 k + 1
- **Isosceles Triangle (Pg.26):** A triangle is isosceles if two of its sides have equal length.
- **Rational Number (Pg.26):** A real number r for which there are integers p and q with q =/= 0 such that r = p/q.
- **Equivalent (Pg.26):** Two statements A and B are equivalent if and only if " A implies B " and " B implies A ."
- **AND (Pg.26):** For two statements A and B , the statement A AND B (written A $\wedge$B)is true when both A and B are true, and false otherwise.
- **OR (Pg.26):** For two statements A and B , the statement A OR B (written A $\vee$B) is false when A is false and B is false, and true otherwise.
- **Overlapping Notation(Pg.28):** That is,when the same symbol is used in both sets.
- **Matching up the Notation(Pg.28):**
- **Previous Knowledge(Pg.30):** Previously prove implication
- **Pythagorean Theorem (Pg.32):** If ABC is a right triangle with sides of lengths a and b and hypotenuse of length c , then $a^2 + b^2 = c^2$.
- **Proposition (Pg.32):** A true statement of interest that you are trying to prove.
- **Theorem (Pg.33):** An important proposition.
- **Lemma(Pg.33):** A proposition that is used in the proof of a subsequent theorem.
- **Corollary (Pg.33):** A proposition whose truth follows almost immediately from a theorem.
- **Axioms (Pg.33):** A statement whose truth is accepted without a proof. Pg.33
- **Contrapositive Statement (Pg.33):** The contrapositive of the statement " A implies B " is the statement " NOT B implies NOT A ."
- **Converse Statement (Pg.33):** The converse of the statement " A implies B " is the statement " B implies A ."
- **Inverse Statement (Pg.33):** The inverse of the statement " A implies B " is the statement " NOT A implies NOT B ."

Chapter 4

- **Quantifier (Pg.41):** One of the two groups of keywords "there is" ("there are,""there exists") and "for all" ("for each," "for every," "for any").
- **Existential Quantifier (Pg.41):** The key words "there is" ("there are," "there exists").
- **Universal Quantifier (Pg.41):** The key words "for all" ("for each," "for every," "for any").
- **Square Integer (Pg.42)r:** An integer n such that there is an integer k with $n = k^2$.
- **Standard Form for "there is" (Pg.42):** There is an object with a certain property,

something happens.

- **Construction Method (Pg.44):** A technique for proving that there is an object with a certain property such that something happens. To do so, construct, guess,produce, or devise an algorithm to produce the desired object. Then show that the object you constructed has the certain property and satisfies the something that happens.

Chapter 5

- **Set (Pg.53):** A collection of items.
- **Member/Element of a Set (Pg.53):** An item that belongs to a given set.
- **Set-Builder Notation (Pg.54):** Using a verbal and mathematical description for the members of the set. The set of all real numbers that are greater than or equal to 0 is written as follows: $S = \{ \text{real numbers } x : x \geq 0 \}$, where the “ : ” stands for the words “such that.” Everything following the“:” is referred to as the **defining property of the set**.
- **Empty (Pg.54):** The set with no elements, written □.
- **Subset (Pg.55):** A set S is a subset of a set T (written $S \subseteq T$ or $S \subset T$) if and only if, for every element $x \in S$, $x \in T$.
- **Standard Form for “for all/ for every/ for each/ for any” (Pg.55):** For all objects with a certain property, something happens.
- **Equal Sets (Pg.55):** Two sets S and T are equal (written $S = T$) if and only if S is a subset of T and T is a subset of S.
- **Choose Method (Pg.56):** A technique for proving that, for every object with a cer-tain property, something happens. To do so, choose a generic object with the certain property. Then show that, for this chosen object, the something happens.
- **Maximizer (Pg.63) :**The real number x^* is a maximizer of the function f if and only if for every real number x , $f(x) \leq f(x^*)$.
- **On the set (Pg.63):** Suppose that f and g are functions of one variable. Then $g \geq f$ on the set S of real numbers if and only if for every element $x \in S$, $g(x) \geq f(x)$.
- **Strictly Increasing (Pg.63):** A function f of one real variable is strictly increasing if and only if for all real numbers x and y with $x < y$, $f(x) < f(y)$.
- **Convex Set(Pg.63)**: A set C of real numbers such that, for all elements $x,y \in C$ and for all real numbers t with $0 \leq t \leq 1$, $tx + (1 - t) y \in C$.
- **Convex Function (Pg.63)**: A function f of one variable such that, for all real numbers x and y and for all real numbers t with $0 \leq t \leq 1$, it follows that $f(tx + (1 - t) y) \leq tf(x) + (1 - t) f(y)$.

Chapter 6

- **Specialization(Pg.69) :** The result of replacing mathematical symbols in a generaliza-tion with specific values.
- **Upper Bound for a Set (Pg.71):** A real number u such that, for all $x \in S$, $x \leq u$ (where S is a given set of real numbers).
- **Least Upper Bound (Pg.73):** A real number u such that, for a given set S of real

numbers, (1) u is an upper bound for S and (2) for every upper bound v for S , $u \le v$.

Chapter 7

- **Syntax Error (Pg.83):** A mistake in a mathematical sentences in which the symbols or operations make no sense or cannot be performed.
- **Onto/Surjective Function (Pg.84):** A real-valued function f of one real variable such that, for every real number y , there is a real number x such that $f(x) = y$.
- **Bounded Above Function (Pg.87):** A real-valued function f of one real variable for which there is a real number y such that, for every real number x , $f(x) \le y$.
- **Bounded (Pg.87):** b. A set of real numbers S is bounded if and only if there is a real number $M > 0$ such that, $\forall$ element $x \in S$, $|x| < M$.
- **Continuous Function at the Point (Pg.87):** A function f of one variable such that,at a given point x , for every real number □ > 0, there is a real number $\delta > 0$ such that, for all real numbers y with $|x - y| < \delta$, $|f(x) - f(y)| < \epsilon$.
- **Converges to (Pg.87):** Suppose that x 1 ,x 2 ,... and x are real numbers. The sequence x_1 ,x_2 ,... converges to x if and only if $\forall$ real numbers $\epsilon > 0$, $\exists$ an integer $j \ge 1$ $\ni$ $\forall$ integer k with $k > j$, $|x_k - x| < \epsilon$
- **Linear function(Pg.92):** A real-valued function f of one real variable for which there are real numbers m and b such that, for all real numbers x, $f(x) = mx + b$.

Chapter 8

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- **Increasing (Pg.98):** A sequence x_1 ,x_2 ,... of real numbers is decreasing if and only if for every integer k = 1 , 2 ,... , $x_k < x_{k+1}$
- **Decreasing (Pg.98):** A sequence x_1 ,x_2 ,... of real numbers is decreasing if and only if for every integer k = 1 , 2 ,... , $x_k > x_{k+1}$
- **Strictly Monotone (Pg.99):** A sequence x_1 ,x_2 ,... of real numbers is strictly monotone if and only if the sequence is increasing or decreasing. (Use the word "and" in the negation.)
- **Greatest Common Divisor (Pg.99):** An integer d is the greatest common divisor of the integers a and b if and only if (i) d | a and d | b and (ii) whenever c is an integer for which c | a and c | b , it follows that c | d .
- **Derivative (Pg.99):** The real number $f(\bar{x})$ is the derivative of the function f at the point $\bar{x}$ if and only if $\forall$ real number > 0, $\exists$ a real number $\delta > 0$ such that $\forall$ real number x with $0 < |x - \bar{x}| < \delta$, $|(f(x) - f(\bar{x})) / (x - \bar{x})| <$ □.

Chapter 9

- **Contradiction Method (Pg. 102):** A technique for proving that " A implies B " in which you work forward from the assumption that A and NOT B are true to reach a contradiction to some statement that you know is true.

- **One-to-One (Pg.111:** A real-valued function f of one real variable such that, for all real numbers x and y with x =/= y , f (x) = /= f (y).

Chapter 10

- **Contrapositive Method (Pg.115):** A technique for proving that “ A implies B ” in which you prove that “ NOT B implies NOT A ” by working forward from NOT B and backward from NOT A .

Chapter 11

- **Uniqueness Method (Pg. 125):** A technique used when the proposition contains the keyword “unique” (or “one and only one” or “exactly one”) as well as the quantifier “there is”. There are three different forms for the Uniqueness Method: forward uniqueness, direct backward uniqueness, and indirect backward uniqueness.

Chapter 12

- **Induction Method (Pg. 133):** A proof technique used in the special case when the conclusion of the proposition involves a “for all” such as “for every integer n>=1, `something happens’”.