Chapter 6 Solutions

3) a) x^* is a maximizer of f
   i) look for a specific real number, y, to which the specialization applies
   ii) no additional property
   iii) conclude that $f(y) \le f(x^*)$

b) $g \ge f$ on S
   i) look for a specific element, y, to which the specialization applies
   ii) show that $y \in S$
   iii) conclude that $g(y) \ge f(y)$

c) u is an upper bound for S
   i) look for a specific element, y, to which the specialization applies
   ii) show that $y \in S$
   iii) conclude that $y \le u$

5) a) No. The quantifiers are "there exists", not "for all".
b) No. While the quantifier is "for all", it's in the hypothesis.
c) Yes. There is a "for all" quantifier in the hypothesis.
d) Yes. There is a "for all" quantifier in the hypothesis.

6) a) The number, m, must be prime.
b) The real number, y, must be positive and in the set S
c) The side AB must have length c such that $c^2=2m^2-2m^2\cos(C)$.
d) The two sides CD and EC of triangle CDE are parallel to the two sides FD and DA of triangle FDA.

7) a) $a = b = X$: $\sin(2X)=\sin(X + X)=\sin(X)\cos(X)+\cos(X)\sin(X) = 2\sin(X)\cos(X)$
b) $A = S^c$ and $B = T^c$: $(A \cup B)^c = (S^c \cup T^c)^c = (S^c)^c \cap (T^c)^c = A^c \cap B^c$

8) a) $x=0$, $y=1$, $t=1/2$:
Note that $tx+(1-t)y=(1/2)0+(1-1/2)1=1/2$ so that $f(tx+(1-t)y) \le tf(x)+(1-t)f(y)$ becomes $f(1/2) \le 1/2f(0) + (1-1/2)f(1) = (f(0)+f(1))/2$.
b) $c = (a+b)/2$, $d = (a-b)/2$: Note $c^2 - d^2= (a^2+2ab+b^2)/4 - (a^2-2ab+b^2)/4 = ab \ge 0$ so $c^2 \ge d^2$. Then $(c^2 - d^2)^{1/2} \le c$ becomes $(ab)^{1/2} \le (a+b)/2$.

13) Proof: Let x in R be chosen arbitrarily. Since $R \subseteq S$, we know $x \in S$. Turning to the second hypothesis, $x \in T$. In the same way, $S \subseteq T$ implies $x \in T$. Since this argument works for any x we have successfully proven: $R \subseteq T$.

18) Proof: Let $x \in S$ be chosen arbitrarily. By definition of upper bound, $x \le u$. But, $u \le v$, and so $x \le v$. Since this holds for any x S, we know $x \le v$ for all x S. This is what it means for v to be an upper bound for S.

19) In online solution.
23) In online solution.
24) Discussed in class.