Chapter 4 Solutions

Math 300 - Spring 2014

1. c) Object: a point (x,y)
Certain Property: $x \geq 0$, $y \geq 0$
Something Happens: $y= m_1x+b_1$ and $y= m_2x+b_2$

e) Object: integers m and n
Certain Property: at least one of m and n is not zero and gcd(m,n)=c
Something Happens: am+bn=c

3. a) Show that a positive integer n satisfies the "something happens": $n! > 3^n$
b) Show that an integer p is greater than 1 and satisfies the "something happens": p|n
c) Show that $a_0+a_1x+a_2x^2+ \dots a_nx^n$ has n roots.

4. b) Find an integer solution to f(x)=0.
c) Find the solution(s) to $y= m_1x+b_1$ and $y= m_2x+b_2$, and check that they are non-negative.
e) Find integers m and n such that am + bn = c

6. The construction method is used in this proof in the first sentence: "Because n is an even integer, there is an integer k for which n=2k." Now that k has been introduced in the problem, square this relationship: $n^2=4k^2$ and write $4k^2=2(2k^2) = 2j$ where $j=2k^2$. Since $n^2=2j$, with j an integer, n^2 is even.

9. a)

Positive Integer n	n!	3^n
0	1	1
1	1	3
2	2	9
3	6	27
4	24	81
5	120	243
6	720	729
7	5040	2187

Through trial and error, the smallest positive integer that satisfies $n!>3^n$ is n=7.

b)

P	n	(1+r/100)^ nP
1	1	1.05
1	5	1.2762
1	10	1.6288
1	14	1.9799
1	15	2.0789
1	16	2.1828

By inspection of this table, the principal will double with n=15.

c) Through trial and error using a graphing calculator and the cos(x) function we can see that the angle is between 0 and pi/4 for which the first three decimals are the same: is the angle in decimal form 0.739.

12 The author implicitly assumes log2(r)>0. If log2(r) < 0, then n>1/log2(r) is true for n=0, but then 1/n is not defined.

15. The proof isn't correct. The author uses x for an element in R ∩ S and also, in a similar manner, S ∩ T. The elements should be different names.

16. This proof is not correct. In the printed "proof" there is no correlation between n and p; you can construct p from n.

17. There are two errors in this proof. First, the requirement that m be even is never applied; it should say that m=2k for some integer k. Also, in their calculations, they show $m^2 + n^2$-1=2k, which says that $m^2 + n^2$-1 is even – not divisible by 4.

18. If p=m+1, then there is no integer between m and p, so the first sentence in the proof is not correct.

19. Analysis of Proof: The forward-backward method gives rise to the key question, "How can I show that an integer (namely, a) divides another integer (namely, c)?"
By the definition, one answer is to show that
B1: There is an integer k such that c = ak.
There are two hypotheses:
H1: a|b (there exists an integer m so that b=ma)
H2: b|c (there exists an integer n so that c=nb)
Combining H1 and H2 we determine that
A1: c=nb = n(ma) = (nm)a = ka where k=nm is an integer.
This is exactly what we need to complete the proof.

Proof: Because a|b and b|c, by definition, there are integers p and q for which b=ap and c=bq. But then c=bq=(ap)q=a(pq), where pq is an integer, and so a|c.

22. The goal we seek to fulfill is:
B1: There are integers p and q with q=0 such that s/t=p/q.
There are two hypotheses:
H1: s is rational (e.g. there exist integers a and b, with b<>0, such that s = a/b.
H2: t is rational and t<>0 (e.g. there exist integers c and d with c<>0 and d<>0 such that t = c/d)
Then, continuing from H1 and H2: s/t = (a/b) / (m/n) = an / bm = p/q where p=an is an integer and q=bm and p and q are integers and q is not zero.

Proof: Because s and t are rational, there are integers a, b, c, and d, with b <> 0 and d <> 0, such that s = a/b and t = c/d. Then s/t = ad/bc = k/m, with m<>0 so s/t is rational.