Chapter 2 Answer Key

MATH 300 – Spring 2014

Prepared by Charles Champion

2.7 C is incorrect for the reason being that in the hypothesis we are given a function, not a polynomial.

2.8 A is fine.

B is too specific to the problem, needs to be abstract (should not refer to R, S, and T).

C is also too specific to the problem (should not refer to R, S, and T, or to "greater than or equal to 1").

D is not equivalent to the statement (their question would be appropriate if the conclusion was that ((R intersect S) intersect T) is not empty).

2.11 A is correct only because the problem tells us we are dealing with "nonzero integers"

B is correct

C is incorrect (Counter-example: -2 and 2 are nonzero integers. $-2^2=4$, $2^2=4$, 4=4; -2≠2)

D is correct

2.12 Along with C, A and D is incorrect.

(Counter-example A: 0 and 0 are real numbers. 0/0=DNE, DNE≠1; 0=0.)

(Counter-example D: 2.5 and 2.4 are real numbers. 2.5-2.4=.1, .1≤1; 2.5≠2.4)

2.15.a 1. How do I know if a number is positive?

2. How can I show that a number is less than or equal to another number.

2.15.b 1. How can I show that two lines are perpendicular?

2. How can I prove that the angle between two lines is 90°

2.16.a 1. How can I show two numbers are equal to each other?

2. How can I show that two sides have the same length?

2.16.b 1. How can I show that two sets have a common point?

2. How can I show that a set has at least one element?

2.22 1. How can I show that a number is positive?

2. Show that the quadratic formula provides a positive solution to the given equation.

3. Show that $x=-b/2a$ is positive and a solution to the given equation.

2.23 1. How can I show that a triangle is equilateral?

2. Show that the lengths of all three sides of the triangle are equal to each other.

3. Show that the length of RS equals the length of ST equals the length of RT.

2.27 The forward process used is incorrect, just because R is a subset of S does not mean each element of S is an element of R. S could have elements that are not contained in R. A correct forward step would be "every element of R is also an element of S".

2.30.a 1. Length of AB= length of BC= length of CD= length of AD

2. length of AB^2=Area

2.30.b 1. n^2 is even

2. both n+1 and n-1 are odd.

2.30.c 1. The tangent line and the graph of $y=x^2+x$ have a common point.

2. The lines have the same slope at a common point.

2.31 d is not valid. The hypothesis never states limits for the variable x and when x=5 the right-hand side of the equation in d is not defined (because the denominator equals 0 when x=5).

2.32 c is not valid. No limits are set for the variable n and when n=1 the statement $c^{n-2}>b^{n-2}$ is $1/c > 1/b$, which is false when c>b (because c>b implies $1/c<1/b$).

2.33 The last sentence is incorrect. Taking the square root of both sides of the equation produces $\pm\sqrt{(b^2-4ac)}= b-2a$.

2.35 The key question was "How can I prove a real number is equal to zero?" and the questioned was answered by showing $0 \le x \le 0$.

2.40 Hypothesis: ΔABC is an isosceles right triangle.

Conclusion: $z^2/4$ is the area of the triangle.

H1. $x=y$	Given.
H2. $x-y=0$	Subtract y from both sides.
H3. $(x-y)^2=0$	Square both sides.
H4. $y^2+x^2-2xy=0$	Expand.
H5. $y^2+x^2=2xy$	Add 2xy on both sides.
H6. $z^2=y^2+x^2$	Pythagorean Theorem.
H7. $z^2=2xy$	Substitution.
H8. $z^2/4=xy/2$	Divide both sides by 4.
H9. area(ΔABC) = xy/2	Known fact from geometry
C1. area(ΔABC) = z^2/4	Substitution

Proof: Since ΔXYZ is isosceles then x=y, so x-y=0. When x-y is squared, it produces $y^2+x^2-2xy=0$. Add 2xy on both sides and this produces $y^2+x^2=2xy$. Substitute this formula into the Pythagorean Theorem makes $z^2=2xy$. Divide both sides by 4 to get $z^2/4=xy/2$. Since the area, A, of the triangle is one-half the product of its length and height: $A=xy/2= z^2/4$.■