MATH 300 (Section 001)
Prof. Meade

University of South Carolina
Spring 2014

Exam 3
15 April 2014

Name: ______________________

Instructions:

1. There are a total of 7 problems on 4 pages. Check that your copy of the exam has all of the problems.
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. To be eligible for full credit, explanations and justifications must be written in complete English sentences.

Problem	Points	Score
1	10	
2	20	
3	10	
4	20	
5	10	
6	10	
7	20	
Total	100	

Good Luck!

1. (10 points)

   Definition. A set of real numbers S is **bounded** if and only if there is a real number $M > 0$ such that, $\forall$ elements $x \in S$, $|x| < M$.

   Identify the object, the certain property, and the something that happens for each quantifier as they appear left to right in the above definition.

2. (20 points)

   Proposition: For all real numbers x and y with $x < y$, there is a rational number r such that $x < r < y$.

   Proof. Let x and y be real numbers with $x < y$. Then let $\epsilon = y - x > 0$ and so, from a proposition in Exercise 7.11 (which you proved in the homework), there is an integer $n > 0$ such that $n\epsilon > 2$. Now let m be an integer with $nx < m < ny$. Then the desired rational number is $r = \frac{m}{n}$ and so the proof is complete.

   Answer the following questions about the preceding proof.

   (a) What proof technique is used in the first sentence? Why is this technique chosen? Explain.

   (b) What is being done in the second sentence?

   (c) What proof technique is being used in the last sentence of the proof? Why is this technique being used? Explain.

   (d) Is the claim that the proof is complete in the last sentence justified? Why or why not?

3. (10 points)

   Definition. A sequence x_1, x_2, $\ldots$ of real numbers is **increasing** if and only if for every integer $k = 1, 2, \ldots, x_k < x_{k+1}$.

   Write the negation (NOT) of the following definition in such a way that the word "not" does not appear in the statement after the words "if and only if".

4. (20 points)

   Proposition: Suppose that m and n are integers. If either mn is divisible by 4 or n is not divisible by 4, then n is an even integer and m is an odd integer.

   (a) What statement(s) will you work forward from if you want to prove "NOT B implies NOT A"?

   (b) What statement(s) will you work backward from if you want to prove "NOT B implies NOT A"?

5. (10 points)

   Proposition: If f and g are two functions such that (1) $g \geq f$ and (2) f is unbounded above, then g is unbounded above.

   When applying the contradiction method to prove this proposition, what should you assume?

6. (10 points)

   Proposition: If m does not divide n, then $mx^2 + nx + (n - m)$ has no positive integer root.

   In a proof by the contrapositive method, which of the following is a result of the forward process? Explain your answer.

   (a) m divides n
   (b) There is an integer $x > 0$ such that $mx^2 + nx + (n - m) \neq 0$.
   (c) There is an integer $x > 0$ such that $mx^2 + nx + (n - m) = 0$.
   (d) There is an integer $x \leq 0$ such that $mx^2 + nx + (n - m) = 0$.

7. (20 points)

   Proposition: If n and p are positive integers and $n|p$, then $n \leq p$.

   Proof. Suppose $n > p$. Then because $n|p$, there is an integer c such that $p = cn$. Now $n > 0$ and $p > 0$, so $c > 0$. Therefore, $p = cn > cp \geq p$, and so the proof is complete.

   (a) Is the contradiction or contrapositive proof technique being used?

   (b) If the contradiction method is used, identify the contradiction.