MATH 300 (Section 001)
Prof. Meade

University of South Carolina
Spring 2014

Exam 2
20 March 2014

Name: ______________________

Instructions:

1. There are a total of 7 problems on 6 pages. Check that your copy of the exam has all of the problems.
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. To be eligible for full credit, explanations and justifications must be written in complete English sentences.

Problem	Points	Score
1	15	
2	20	
3	10	
4	10	
5	15	
6	15	
7	15	
Total	100	

Good Luck!

1. (15 points) State the definition of each of the following terms.

   (a) Let m and n be integers. $n|m$ (n divides m) if and only if

   (b) An integer n is even if and only if

   (c) A real number r is rational if and only if

2. (20 points) Rewrite each of the following statements so that the quantifier ("there-is" or "for-all") appears in standard form. Then identify the object, the certain property, and the something that happens.

   (a) If θ is an angle, then $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$.

   (b) The equation $f(x) = 3$ has a solution in the interval $[1, 4)$.

   (c) Some element of the set S is < 2.

   (d) The square root of the product of two nonnegative real numbers p and q is not less than the average of the two numbers.

   (e) The integer $n > 1$ can be divided by some integer p with $1 < p < n$.

3. (10 points) For the following "for-all" statements, what properties must the given object satisfy so that you can apply specialization? Given that the object does satisfy those properties, what can you conclude about the object?

Statement: If a and b are real numbers with $a < 0$ and $y = -b/(2a)$, then for all real numbers x, $ax^2 + bx \leq ay^2 + by$.

Given object: $4x - x^2$

4. (10 points) To what specific object would you specialize the following "for-all" statement so that the result of specialization leads to the desired conclusion. Verify that the object to which you are applying specialization satisfies the certain property in the "for-all" statement so that you can apply specialization.

Statement: f is a function of one real variable such that, for all x, y, and t with $0 \leq t \leq 1$, $f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$.

Desired conclusion: the function satisfies $f(u) \leq f(0) + u(f(1) - f(0))$ for all $u \in [0, 1]$.

5. (15 points) Consider the following statement:

   if a, b, and c are integers with $a|(b+c)$ and $a|b$, then $a|c$.

   (a) What method (construction, choose, or specialization) would be used in the proof of this statement?

   (b) Prove the statement.

6. (15 points) Consider the following statement:

   if a and b are real numbers, then the set $C = \{\text{real numbers } x\colon\ ax < b\}$ is a convex set.

   (a) What method (construction, choose, or specialization) would be used in the proof of this statement?

   (b) Prove the statement.

Recall that a set S is convex if and only if for all $x, y \in S$ and all $0 \leq t \leq 1$, $tx + (1-t)y \in S$.

7. (15 points) Consider the following statement:

   For real numbers L and M, if L is a lower bound for a set of real numbers S and $L \geq M$, then M is a lower bound for S.

   (a) What method (construction, choose, or specialization) would be used in the proof of this statement?

   (b) Prove the statement.

A number L is a lower bound for a set S if and only if $L \leq x$ for all $x \in S$.