MATH 300 (Section 001)
Prof. Meade

University of South Carolina
Spring 2014

Exam 1
18 February 2014

Name: ______________________________

Instructions:

1. There are a total of 9 problems on 6 pages. Check that your copy of the exam has all of the problems.
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and should not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. To be eligible for full credit, explanations and justifications must be written in complete English sentences.

Problem	Points	Score
1	20	
2	8	
3	12	
4	8	
5	12	
6	12	
7	12	
8	8	
9	8	
Total	100	

Good Luck!

1. (20 points) Prepare a truth table for each of the following statements.

   (a) $\sim (A \vee B)$

   (b) $(\sim A) \wedge (\sim B)$

   (c) $((\sim B) \wedge (A \Rightarrow B)) \Rightarrow \sim A$

   (d) In general, how can you tell if two statements are logically equivalent?

   (e) Which, if any, of the above statements are logically equivalent?

2. (8 points) Prove that the following statement is false: *If n is prime, then $n^2 + n + 7$ is prime.*
   HINT: *Find a counterexample.*

3. (12 points) Write the indicated statement for each of the following propositions.

   (a) The converse of "If n is an integer for which n^2 is odd, then n is odd."

   (b) The contrapositive of "If r is a real number such that $r^2 = 2$, then r is not rational."

   (c) The inverse of "If $n > 1$ is an integer for which $2^n - 1$ is prime, then n is prime".

4. (8 points) Suppose you know that

   Proposition. *If the right triangle RST with sides of lengths r and s and hypotenuse of length t satisfies $t = \sqrt{2rs}$, then the triangle RST is isosceles.*

   What would you have to show in order to use the above proposition to prove "If the right triangle ABC with side of lengths a and b and hypotenuse of length c has an area of $c^2/4$, then the triangle is isosceles."

5. (12 points) For each of the following statements, obtain a new statement in the backward process by using a definition to answer the key question. If necessary, rewrite the definitions so that there is no overlapping notation.

   (a) The integer $m > 1$ is prime.

   (b) triangle ABC is equilateral

   (c) $\sqrt{n}$ is rational (n is an integer)

6. (12 points) Use a definition to work forward one step from the hypothesis.

   (a) If n is an integer greater than 1 for which $2^n - 1$ is prime, then n is prime.

   (b) If a, b, and c are integers for which $a \mid b$ and $b \mid c$, then $a \mid c$.

   (c) If the quadrilateral $ABCD$ is a parallelogram with one right angle, then $ABCD$ is a rectangle.

7. (12 points) Consider the following statement:

If n is an odd integer, then $n^2 + 1$ is an even integer.

   (a) Pose a key question.

   (b) Then use a definition to answer the question abstractly.

   (c) And, finally, use that same definition to apply the answer to the specific problem.

8. (8 points) Prove: *Let a and b be integers. If $a \mid b$, then $a^2 \mid b^2$.*

9. (8 points) Provide an analysis of the following proof.

   If n is an integer greater than 2, a and b are the lengths of the legs of a right triangle, and c is the length of the hypotenuse, then $c^n > a^n + b^n$.

   Proof. You have that $c^n = c^2c^{n-2} = (a^2+b^2)c^{n-2}$. Observing that $c^{n-2} > a^{n-2}$ and $c^{n-2} > b^{n-2}$, it follows that $c^n > a^2(a^{n-2}) + b^2(b^{n-2})$. Consequently, $c^n > a^n + b^n$. ∎