1. (15 points) Evaluate the iterated integral $\int_{-1}^{3}\int_{0}^{1} ye^{xy}\,dx\,dy$.

$$\int_{-1}^{3}\int_{0}^{1} ye^{xy}\,dx\,dy = \int_{-1}^{3}\int_{0}^{y} e^{u}\,du\,dy = \int_{-1}^{3} e^{y}-1\,dy = \left(e^{y}-y\right)\Big|_{-1}^{3} = \left(e^{3}-3\right)-\left(e^{-1}+1\right) = \boxed{e^{3}-e^{-1}-4.}$$

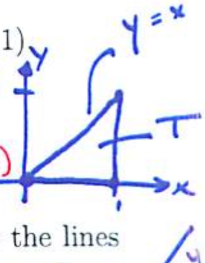
$u = xy$, $du = y\,dx$

2. (30 points) Write each integral as an equivalent iterated integral that would be reasonable for you to evaluate. Do not evaluate any integrals.

(a) $\iint_T \frac{1}{1+x^2}\,dA$ where T is the triangular region with vertices $(0,0)$, $(1,0)$, $(1,1)$.

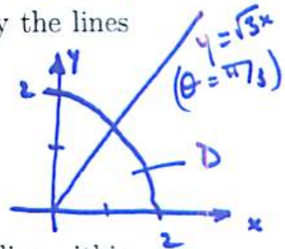
$$= \boxed{\int_0^1\int_0^x \frac{1}{1+x^2}\,dy\,dx} \quad \text{or} \quad \int_0^1\int_y^1 \frac{1}{1+x^2}\,dx\,dy$$

but this one will be a lot harder to evaluate. (try it for yourself)



(b) $\iint_D \cos(x^2+y^2)\,dA$ where D is the region in the first quadrant bounded by the lines $y=0$ and $y=\sqrt{3}x$ and the circle $x^2+y^2=4$.

$$= \boxed{\int_0^{\pi/3}\int_0^2 \cos(r^2)\; r\,dr\,d\theta}$$



3. (15 points) Find an iterated (triple) integral for the volume V of the solid that lies within the sphere $x^2+y^2+z^2=4$, above the xy-plane, and below the cone $z=\sqrt{x^2+y^2}$. Do not evaluate any integrals.

$\rho\cos\varphi = \rho\sin\varphi$
$\cos\varphi = \sin\varphi$
$\varphi = \pi/4$

$$V = \iiint_E dV = \boxed{\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^2 \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta}$$

4. (16 points) Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx$ $\overset{\text{cylindrical coord.}}{=} \int_0^{2\pi}\int_0^{2}\int_r^{2} r^2\cdot r\,dz\,dr\,d\theta$

circle w/ radius 2; $z = r$; r^2

$$= \int_0^{2\pi}\int_0^2 r^3(2-r)\,dr\,d\theta = \int_0^{2\pi}\left(\frac{1}{2}r^4-\frac{1}{5}r^5\right)\Big|_0^2 d\theta = \int_0^{2\pi} 8-\frac{32}{5}\,d\theta = \frac{8}{5}\int_0^{2\pi} d\theta = \frac{8}{5}\cdot 2\pi = \boxed{\frac{16}{5}\pi.}$$

5. (24 points)

(a) Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = xy\mathbf{i}+3y^2\mathbf{j}$ and $\mathbf{r}(t) = 11t^4\mathbf{i}+t^3\mathbf{j}$ for $0\le t\le 1$.

$\vec{F}(\vec{r}(t)) = \langle 11t^7, 3t^6\rangle$ $\qquad d\vec{r} = \langle 44t^3, 3t^2\rangle.$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 484t^{10}+9t^8\,dt = \left(44t^{11}+t^9\right)\Big|_0^1 = (44+1)-0 = \boxed{45.}$$

(b) Evaluate $\int_C 2x\,ds$ where C consists of the arc C_1 of the parabola $y=x^2$ from $(0,0)$ to $(1,1)$ followed by the vertical line segment C_2 from $(1,1)$ to $(1,2)$.

C_1: $\vec{r}_1(t) = \langle t, t^2\rangle \quad 0\le t\le 1$
$\vec{r}_1'(t) = \langle 1, 2t\rangle$
$|\vec{r}_1'(t)| = \sqrt{1+4t^2}$

$$\int_{C_1} 2x\,ds = \int_0^1 2t\cdot\sqrt{1+4t^2}\,dt = \frac{2}{3}\left(1+4t^2\right)^{3/2}\cdot\frac{1}{4}\Big|_0^1 = \frac{1}{6}\left(5^{3/2}-1\right)$$

C_2: $\vec{r}_2(t) = \langle 1, 1+t\rangle \quad 0\le t\le 1$
$\vec{r}_2'(t) = \langle 0, 1\rangle$
$|\vec{r}_2'(t)| = 1.$

$$\int_{C_2} 2x\,ds = \int_0^1 2\cdot 1\,dt = \int_0^1 2\,dt = 2.$$

$$\text{so } \int_C 2x\,ds = \int_{C_1} 2x\,ds + \int_{C_2} 2x\,ds = \frac{1}{6}5^{3/2}-\frac{1}{6}+2 = \boxed{\frac{1}{6}5^{3/2}+\frac{11}{6}.}$$