MATH 241 (Section 502)
Prof. Meade

University of South Carolina
Fall 2010

Exam 3
22 November 2010

Name: Key
SS # (last 4 digits): ______

Instructions:

1. There are a total of 4 problems
2. No electronic or other inanimate objects can be used during this exam. All questions have been designed with this in mind and do not involve unreasonable manual calculations.
3. Be sure you answer the questions that are asked.
4. Copy your final answer to each question to the back of this page.
5. You must show all of your work to receive full credit for a correct answer. Correct answers with no supporting work will be eligible for at most half-credit.
6. Your answers must be clearly labeled and written legibly on additional sheets of paper (that I will provide). Be sure to put your name on each sheet, include the question number for all work, and staple all pages — in order — to this page when you turn in your completed test.
7. Check your work. If I see *clear evidence* that you checked your answer (when possible) and you *clearly indicate* that your answer is incorrect, you will be eligible for more points than if you had not checked your work.

Problem	Points	Score
1	12	
2	12	
3	51	
4	25	
Total	100	

Happy Thanksgiving!

1. (12 points) Values for a function $f(x, y)$ defined on $R = [1,5] \times [2,6]$ are given in the table below.

$x \longrightarrow$

$y \downarrow$

y \ x	1	2	3	4	5
2	2	0	-3	-6	-5
3	3	1	-4	-8	-6
4	4	3	0	-5	-8
5	5	5	3	-1	-4
6	7	8	6	3	0

Estimate $\iint_R f(x, y)\, dA$ with a Riemann sum with $m = 2$, $n = 2$, and take the sample points to be the upper-right corner of each square.

$$\iint_R f(x,y)\, dA \approx \big(f(3,4) + f(5,4) + f(3,6) + f(5,6)\big)\, \Delta A = (0 + -8 + 6 + 0) \cdot 4 = -8$$

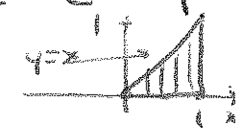
2. (12 points) Calculate the iterated integral $\int_0^1 \int_{-1}^3 y e^{xy}\, dy\, dx$.

interchange the order of integration:

$$\int_0^1 \int_{-1}^3 y e^{xy}\, dy\, dx = \int_{-1}^3 \int_0^1 y e^{xy}\, dx\, dy = \int_{-1}^3 e^{xy}\Big|_{x=0}^{x=1} dy = \int_{-1}^3 e^y - 1\, dy = (e^y - y)\Big|_{-1}^3 = e^3 - 3 - (e^{-1} + 1) = e^3 - e^{-1} - 4$$

3. (51 points) Evaluate each definite integral.

(a) $\iint_T \frac{1}{1+x^2}\, dA$ where T is the triangular region with vertices $(0,0)$, $(1,0)$, $(1,1)$.



$$= \int_0^1 \int_0^x \frac{1}{1+x^2}\, dy\, dx = \int_0^1 \frac{y}{1+x^2}\Big|_0^x dx = \int_0^1 \frac{x}{1+x^2}\, dx = \frac{1}{2}\ln(1+x^2)\Big|_0^1 = \frac{1}{2}\ln(2) - \frac{1}{2}\ln(1) = \frac{1}{2}\ln 2$$

(b) $\iint_D (x^2 + y^2)^{3/2}\, dA$ where D is the region in the first quadrant bounded by the lines $y = 0$ and $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$.

$$= \int_0^{\pi/3} \int_0^3 (r^2)^{3/2}\, r\, dr\, d\theta = \int_0^{\pi/3} \int_0^3 r^4\, dr\, d\theta = \int_0^{\pi/3} \frac{1}{5} r^5 \Big|_0^3 d\theta = \int_0^{\pi/3} \frac{3^5}{5}\, d\theta = \frac{3^5}{5}\theta\Big|_0^{\pi/3} = \frac{3^5}{5} \cdot \frac{\pi}{3} = \frac{3^4 \pi}{5} = \frac{81\pi}{5}$$

(c) $\iiint_E yz\, dV$ where E lies above the plane $z = 0$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 9$. use cylindrical coordinates Note: $y > 0$

$$= \int_0^{\pi} \int_0^3 \int_0^{r\sin\theta} r\sin\theta\, z\, r\, dz\, dr\, d\theta = \int_0^{\pi} \int_0^3 r^2 \sin\theta\, \frac{1}{2} z^2 \Big|_{z=0}^{z=r\sin\theta} dr\, d\theta = \frac{1}{2}\int_0^{\pi}\int_0^3 r^4 \sin^3\theta\, dr\, d\theta = \frac{1}{2}\int_0^{\pi} \sin^3\theta\, d\theta \int_0^3 r^4\, dr$$

$$= \frac{1}{2}\left(-\frac{1}{3}\cos\theta\,(2 + \sin^2\theta)\right)\Big|_{\theta=0}^{\pi} \left(\frac{1}{5} r^5\right)\Big|_{r=0}^{3}$$

$$= \frac{1}{2}\left(+\frac{2}{3} - -\frac{2}{3}\right)\left(\frac{1}{5} 3^5\right) = \frac{1}{2} \cdot \frac{4}{3} \cdot \frac{3^5}{5} = \frac{2 \cdot 3^4}{5} = \frac{162}{5}$$

4. (25 points) Let S be the solid under the paraboloid $z = x^2 + 4y^2$ and above the rectangle $R = [0,2] \times [1,4]$.

(a) Express the volume of S as an iterated (triple) integral.

use Cartesian (rectangular) coordinates.

$$V = \iiint_S dV = \int_0^2 \int_1^4 \int_0^{x^2+4y^2} dz\, dy\, dx$$

(b) Evaluate the integral found in (a).

$$V = \int_0^2 \int_1^4 x^2 + 4y^2\, dy\, dx = \int_0^2 \left(x^2 y + \frac{4}{3} y^3\right)\Big|_{y=1}^{y=4} dx = \int_0^2 \left(4x^2 + \frac{4}{3}\cdot 4^3\right) - \left(x^2 + \frac{4}{3}\right) dx$$

$$= \int_0^2 3x^2 + 84\, dx = (x^3 + 84x)\Big|_0^2 = 8 + 168 = 176$$