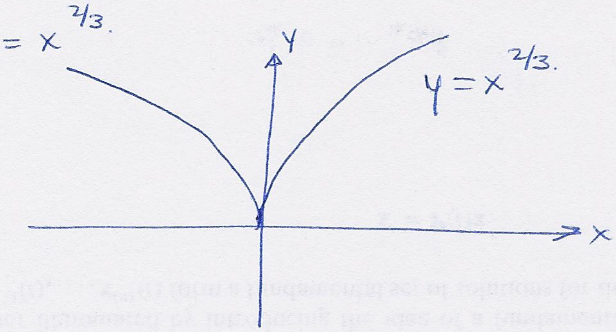
# HW Sol'n for §8.1

a) $y^3 = x^2 \iff y = x^{2/3}$.




Note: To get this plot from Maple you can't use $x^\wedge(2/3)$.
You need to use surd$(x^2, 3)$.
I'll explain to you if you are interested. Ask me!

b) $L = \int_0^1 \left(1 + \left((x^{2/3})'\right)^2\right)^{1/2} dx$ ⟵ using $y = x^{2/3}$

$= \int_0^1 \left(1 + \left(\frac{2}{3}x^{-1/3}\right)^2\right)^{1/2} dx$

$= \int_0^1 \left(1 + \frac{4}{9}x^{-2/3}\right)^{1/2} dx$

$= \int_0^1 \left(1 + \frac{4}{9x^{2/3}}\right)^{1/2} dx$

$= \int_0^1 \left(\frac{9x^{2/3}+4}{9x^{2/3}}\right)^{1/2} dx$

$= \int_0^1 \frac{(9x^{2/3}+4)^{1/2}}{3x^{1/3}} dx$ ⟵ because of this $x^{1/3}$ in the denominator, this is an improper integral

$= \lim_{A\to 0^+} \int_A^1 \frac{(9x^{2/3}+4)^{1/2}}{3x^{1/3}} dx$

$u = 9x^{2/3} + 4$

$du = 9\left(\frac{2}{3}\right)x^{-1/3} dx$

$\frac{1}{6} du = x^{-1/3} dx$

$= \lim_{A\to 0^+} \int_{x=A}^{x=1} \frac{u^{1/2}}{3\cdot 6} du$

$= \lim_{A\to 0^+} \frac{1}{18}\cdot\frac{2}{3} u^{3/2} \Big|_{x=A}^{x=1}$

$= \lim_{A\to 0^+} \frac{1}{27}(9x^{2/3}+4)^{3/2} \Big|_A^1$

$= \lim_{A\to 0^+} \frac{1}{27}\left(13^{3/2} - (9A^{2/3}+4)^{3/2}\right)$

$= \frac{1}{27}\left(13^{3/2} - 4^{3/2}\right)$

$= \frac{13^{3/2} - 8}{27}$

Now, using $x = y^{3/2}$ :

$$L = \int_0^1 \sqrt{\left((y^{3/2})'\right)^2 + 1}\ dy$$

$$= \int_0^1 \left(\left(\tfrac{3}{2}y^{1/2}\right)^2 + 1\right)^{1/2} dy$$

$$= \int_0^1 \left(\tfrac{9}{4}y + 1\right)^{1/2} dy$$

$$u = \tfrac{9}{4}y + 1$$

$$du = \tfrac{9}{4}dy$$

$$= \int_1^{13/4} u^{1/2}\left(\tfrac{4}{9}\right) du$$

$$= \tfrac{4}{9}\ \tfrac{2}{3}\ u^{3/2}\Big|_1^{13/4}$$

$$= \tfrac{8}{27}\left(\left(\tfrac{13}{4}\right)^{3/2} - 1\right) = \tfrac{8}{27}\left(\tfrac{13^{3/2}}{8} - 1\right) = \tfrac{13^{3/2}}{27} - \tfrac{8}{27}.$$

(c) From $(-1,1)$ to $(8,4)$, this curve passes through the origin.

We divide this curve into 2 pieces, from $(-1,1)$ to $(0,0)$ and from $(0,0)$ to $(8,4)$.

The length from $(-1,1)$ to $(0,0)$ is the same as from $(0,0)$ to $(1,1)$, i.e. $L_1 = \frac{13^{3/2} - 8}{27}$.

From $(0,0)$ to $(8,4)$, we use the second approach (above: $x = y^{3/2}$, $0 \le y \le 4$:

$$L_2 = \int_0^4 \sqrt{\left((y^{3/2})'\right)^2 + 1}\ dy = \int_0^4 \left(\tfrac{9}{4}y + 1\right)^{1/2} dy = \tfrac{4}{9}\int_1^{10} u^{1/2}\, du$$

$$u = \tfrac{9}{4}y + 1$$

$$du = \tfrac{9}{4}dy$$

$$= \tfrac{4}{9}\ \tfrac{2}{3}\ u^{3/2}\Big|_1^{10}$$

$$= \tfrac{8}{27}\left(10^{3/2} - 1\right) =$$

Thus: $L = L_1 + L_2 = \frac{1}{27}\left(13^{3/2} - 8\right) + \frac{8}{27}\left(10^{3/2} - 1\right) = \frac{8}{27}\left(\frac{13^{3/2}}{8} + 10^{3/2} - 2\right) \approx 10.513$