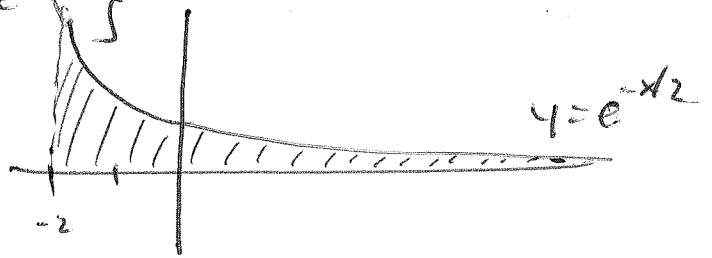


HW Soln for § 7.8

#42. $S = \{(x, y) : x \geq -2, 0 \leq y \leq e^{-x/2}\}$



$$\begin{aligned}
 A &= \int_{-2}^{\infty} e^{-x/2} dx = \lim_{B \rightarrow \infty} \int_{-2}^B e^{-x/2} dx \\
 &= \lim_{B \rightarrow \infty} \left(-2e^{-x/2} \Big|_{-2}^B \right) \\
 &= \lim_{B \rightarrow \infty} \left(-2e^{-B/2} + 2e^{-(-2)/2} \right) \\
 &= 0 + 2e \\
 &= \underline{\underline{2e}}
 \end{aligned}$$

#55. $\int_0^{\infty} \frac{dx}{\sqrt{x}(1+x)} = \int_0^1 \frac{dx}{\sqrt{x}(1+x)} + \int_1^{\infty} \frac{dx}{\sqrt{x}(1+x)}$

Note: $\int \frac{dx}{\sqrt{x}(1+x)} = 2 \int \frac{du}{2x^{1/2}(1+u^2)} = 2 \int \frac{du}{1+u^2} = 2 \arctan u + C = 2 \arctan(\sqrt{x}) + C$

$$\begin{aligned}
 u &= x^{1/2} \\
 du &= \frac{1}{2} x^{-1/2} dx \\
 &= \frac{1}{2} \frac{dx}{x^{1/2}}
 \end{aligned}$$

$$\begin{aligned}
 \int_0^1 \frac{dx}{\sqrt{x}(1+x)} &= \lim_{A \rightarrow 0^+} \int_A^1 \frac{dx}{\sqrt{x}(1+x)} = \lim_{A \rightarrow 0^+} \left(2 \arctan \sqrt{x} \Big|_A^1 \right) \\
 &= \lim_{A \rightarrow 0^+} \left(2 \arctan(1) - 2 \arctan(\sqrt{A}) \right) = 2 \cdot \frac{\pi}{4} - 0 = \frac{\pi}{2}
 \end{aligned}$$

$$\begin{aligned}
 \int_1^{\infty} \frac{dx}{\sqrt{x}(1+x)} &= \lim_{B \rightarrow \infty} \int_1^B \frac{dx}{\sqrt{x}(1+x)} = \lim_{B \rightarrow \infty} \left(2 \arctan \sqrt{x} \Big|_1^B \right) \\
 &= \lim_{B \rightarrow \infty} \left(2 \arctan \sqrt{B} - 2 \arctan(1) \right) = 2 \cdot \frac{\pi}{2} - 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}
 \end{aligned}$$

So $\int_0^{\infty} \frac{dx}{\sqrt{x}(1+x)} dx = \frac{\pi}{2} + \frac{\pi}{2} = \pi$