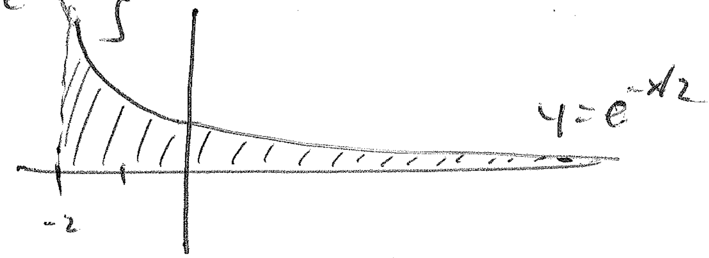
# HW Soln for §7.8

#42. $S = \{(x,y) : x \geq -2,\ 0 \leq y \leq e^{-x/2}\}$




$$A = \int_{-2}^{\infty} e^{-x/2}\,dx = \lim_{B\to\infty} \int_{-2}^{B} e^{-x/2}\,dx$$

$$= \lim_{B\to\infty} \left( -2e^{-x/2} \Big|_{-2}^{B} \right)$$

$$= \lim_{B\to\infty} -2e^{-B/2} + 2e^{+1}$$

$$= 0 + 2e$$

$$= \underline{\underline{2e}}$$

#55. $$\int_0^{\infty} \frac{dx}{\sqrt{x}(1+x)} = \int_0^{1} \frac{dx}{\sqrt{x}(1+x)} + \int_1^{\infty} \frac{dx}{\sqrt{x}(1+x)}$$

Note: $$\int \frac{dx}{\sqrt{x}(1+x)} = 2\int \frac{dx}{2x^{1/2}(1+u^2)} = 2\int \frac{du}{1+u^2} = 2\arctan u + C$$
$$= 2\arctan(\sqrt{x}) + C$$

$u = x^{1/2}$

$du = \frac{1}{2x^{1/2}}\,dx$

$\cancel{= \frac{1}{2}x^{-1/2}}$

$$\int_0^1 \frac{dx}{\sqrt{x}(1+x)} = \lim_{A\to 0^+} \int_A^1 \frac{dx}{\sqrt{x}(1+x)} = \lim_{A\to 0^+} 2\arctan\sqrt{x}\,\Big|_A^1$$

$$= \lim_{A\to 0^+} \left(2\arctan(1) - 2\arctan(\sqrt{A})\right) = 2\cdot\frac{\pi}{4} - 0 = \frac{\pi}{2}$$

$$\int_1^{\infty} \frac{dx}{\sqrt{x}(1+x)} = \lim_{B\to\infty} \int_1^{B} \frac{dx}{\sqrt{x}(1+x)} = \lim_{B\to\infty} \left(2\arctan\sqrt{x}\right)\Big|_1^B$$

$$= \lim_{B\to\infty} \left(2\arctan\sqrt{B} - 2\arctan(1)\right) = 2\cdot\frac{\pi}{2} - 2\cdot\frac{\pi}{4} = \frac{\pi}{2}$$

So $$\int_0^{\infty} \frac{dx}{\sqrt{x}(1+x)}\,dx = \frac{\pi}{2} + \frac{\pi}{2} = \underline{\pi}$$