HW Sol'n for §7.5

#35. $\int_{-1}^{1} x^8 \sin x \, dx$

This antiderivative could be found using integration by parts (many times) or by using Formulas 84 & 85 (many times).
Alternatively, note that the interval is symmetric.
If the integrand is odd, then the definite integral's value is 0.
Let's check:

$$(-x)^8 \sin(-x) = (-1)^8 x^8 (-\sin x) = (-1)^9 x^8 \sin x = -x^8 \sin x.$$

so the integrand is odd and so $\int_{-1}^{1} x^8 \sin x \, dx = 0$.

#42. $\int \frac{\tan^{-1} x}{x^2} dx$

The presence of the $\tan^{-1}$ suggests the use of integration by parts,
with $u = \arctan x \qquad dv = x^{-2} dx$

$du = \frac{dx}{1+x^2} \qquad v = -x^{-1}$

so $\int \frac{\tan^{-1} x}{x^2} dx = -x^{-1} \arctan x + \int \frac{1}{x(1+x^2)} dx$

Next, using partial fractions: $\frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+C}{1+x^2} = \frac{A(1+x^2)+(Bx+C)x}{x(1+x^2)}$

$$= \frac{(A+B)x^2 + Cx + A}{x(1+x^2)}$$

Equating like coefficients in the numerator: $A+B=0 \Rightarrow B=-A=-1$
$C=0$
$A=1$

so $\int \frac{\tan^{-1} x}{x^2} dx = -\frac{1}{x}\arctan x + \int \frac{1}{x} - \frac{x}{1+x^2} dx = -\frac{\arctan x}{x} + \ln|x| - \frac{1}{2}\ln|1+x^2| + C$