HW §7.2 Solution

#56. a. $\int \sin x \cos x \, dx = -\int u \, du = -\frac{1}{2}u^2 + C = -\frac{1}{2}\cos^2 x + C_1$

$u = \cos x$
$du = -\sin x \, dx$

b. $\int \sin x \cos x \, dx = \int u \, du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2 x + C_2$

$u = \sin x$
$du = \cos x \, dx$

c. $\int \sin x \cos x \, dx = \frac{1}{2}\int 2\sin x \cos x \, dx = \frac{1}{2}\int \sin(2x) \, dx = -\frac{1}{4}\cos(2x) + C_3$

d. $\int \sin x \cos x \, dx = \sin^2 x - \int \sin x \cos x \, dx$

$u = \sin x \quad dv = \cos x \, dx$
$du = \cos x \, dx \quad v = \sin x$

so $2\int \sin x \cos x \, dx = \sin^2 x + C$

$\therefore$ so $\int \sin x \cos x \, dx = \frac{1}{2}\sin^2 x + C_4$

Through the use of trig. identities ($\sin^2 x + \cos^2 x = 1$ and $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$) we can show that these different evaluations all differ by a constant.

#59. (see Maple plot)

$$\int_0^{2\pi} \cos^3 x \, dx = \int_0^{2\pi} (1 - \sin^2 x)\cos x \, dx = \int_{x=0}^{x=2\pi} 1 - u^2 \, du = \left(u - \frac{1}{3}u^3\right)\Big|_{x=0}^{x=2\pi} = \left(\sin x - \frac{1}{3}\sin^3 x\right)\Big|_0^{2\pi} = 0.$$

$u = \sin x$
$du = \cos x \, dx$

if you change the limits to u, you get $\int_0^0 1 - u^2 \, du = 0$.

#65. $v(t) = \sin(\omega t)\cos^2(\omega t)$

$$s(t) = \int_0^t v(t) \, dt = \int_0^t \sin(\omega t)\cos^2(\omega t) \, dt = -\frac{1}{\omega}\int_{u=1}^{u=\cos(\omega t)} u^2 \, du = -\frac{1}{3\omega}u^3\Big|_1^{\cos(\omega t)}$$

$u = \cos(\omega t)$
$du = -\omega \sin(\omega t) \, dt$

$$= -\frac{1}{3\omega}\cos^3(\omega t) - \left(-\frac{1}{3\omega}\right)$$

$$= \frac{1}{3\omega}\left(1 - \cos^3(\omega t)\right).$$