

HW Soln for §6.5

#14. $f(x) = 2 + 6x - 3x^2$

avg. value of $f(x)$ on $[0, b]$ is

$$f_{\text{avg}} = \frac{1}{b-0} \int_0^b f(x) dx$$

$$= \frac{1}{b} \left(2x + 3x^2 - x^3 \right) \Big|_0^b$$

$$= \frac{1}{b} (2b + 3b^2 - b^3) = 2 + 3b - b^2$$

To find when (and if) $f_{\text{avg}} = 3$, we solve:

$$2 + 3b - b^2 = 3$$

$$b^2 - 3b + 1 = 0$$

By the quadratic formula:

$$b = \frac{1}{2} (3 \pm \sqrt{9 - 4})$$

$$= \frac{1}{2} (3 \pm \sqrt{5}).$$

So, when $b = \frac{1}{2} (3 + \sqrt{5})$
& when $b = \frac{1}{2} (3 - \sqrt{5})$ the average value of f on $[0, b]$ is 3.