MATH 142 (Section H01)
Prof. Meade

University of South Carolina
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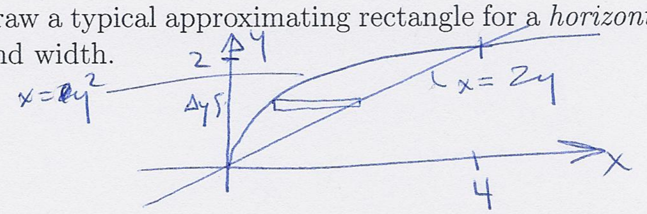
Quiz 4
September 29, 2014

Name: Key
Section: H01

1. (10 points) The curves $y = \sqrt{x}$ and $y = \frac{x}{2}$ enclose a region.

   (a) Sketch the region. Draw a typical approximating rectangle for a *horizontal* slice. Be sure to label the height and width.



width: $2y - y^2$
height: Δy

   (b) Find the area using a definite integral involving horizontal slices.

$$A = \int_0^2 2y - y^2 \, dy$$
$$= \left(y^2 - \tfrac{1}{3}y^3\right)\Big|_0^2$$
$$= \left(4 - \tfrac{8}{3}\right) - 0 = \tfrac{4}{3}$$

   (c) Find the area using a definite integral involving vertical slices.

$$A = \int_0^4 x^{1/2} - \tfrac{x}{2} \, dx$$
$$= \left(\tfrac{2}{3}x^{3/2} - \tfrac{x^2}{4}\right)\Big|_0^4$$
$$= \tfrac{2}{3}4^{3/2} - \tfrac{16}{4} = \tfrac{2}{3}(8) - \tfrac{16}{4} = \tfrac{64-48}{12} = \tfrac{16}{12} = \tfrac{4}{3}$$

   (d) If your answers in (b) and (c) are the same, which approach did you find easier?
   If your answers in (b) and (c) are not the same, which one do you think is more likely to be correct?

(b) was easier for me because it avoided square roots (& 3/2 powers!)

[but (c) is conceptually easier, using $y = f(x)$]