MATH 142 (Section H01)
Prof. Meade

Quiz 3
September 16, 2014

University of South Carolina
Fall 2014

Name: Key
Section: H01

1. (5 points) Write out the form of the partial fraction decomposition of $\dfrac{x^4}{(x+x^3)(x-3)^2}$.
Do not determine the values of the coefficients.

$$\frac{x^4}{(x+x^3)(x-3)^2} = \frac{x^4}{x(1+x^2)(x-3)^2} = \frac{x^3}{(1+x^2)(x-3)^2} = \frac{Ax+B}{1+x^2} + \frac{C}{x-3} + \frac{D}{(x-3)^2}$$

Note: if you do not simplify first, then you'd have one more term in $\frac{E}{x}$ (but you'd find $E=0$).

2. (5 points) Evaluate $\int \frac{27}{25}\frac{1}{x-3} + \frac{27}{10}\frac{1}{(x-3)^3} + \frac{-4x+3}{50(1+x^2)}\,dx$.
your answer

↑ should've been a 2, but I'll work this as stated.

$$\int \frac{27}{25}\frac{1}{x-3} + \frac{27}{10}(x-3)^{-3} - \frac{2}{25}\frac{2x}{1+x^2} + \frac{3}{50}\frac{1}{1+x^2}\,dx$$

$$= \frac{27}{25}\ln|x-3| - \frac{27}{10}\left(\frac{1}{-2}(x-3)^{-2}\right) - \frac{2}{25}\ln(1+x^2) + \frac{3}{50}\arctan x + C$$

$$= \frac{27}{25}\ln|x-3| + \frac{27}{10}(x-3)^{-2} - \frac{2}{25}\ln(1+x^2) + \frac{3}{50}\arctan x + C.$$

QUIZ 3: Extra Credit

$$\frac{x^3}{(x-3)^2(1+x^2)} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{Cx+D}{1+x^2} = \frac{A(x-3)(1+x^2)+B(1+x^2)+(Cx+D)(x-3)^2}{(x-3)^2(1+x^2)}$$

so (equating the numerators)

$$x^3 = A(x-3)(1+x^2)+B(1+x^2)+(Cx+D)(x-3)^2$$

choose $x=3$: $27 = B(1+9) = 10B \Rightarrow \boxed{B = \frac{27}{10}}$

choose $x=0$: $0 = -3A+B+9D \Rightarrow 3A-9D = B = \frac{27}{10}$

$$A - 3D = \frac{9}{10} \quad (1)$$

choose $x=1$: $1 = -4A+2B+4(C+D) \Rightarrow 4A-4C-4D = 2B-1 = \frac{44}{10}$

$$A - C - D = \frac{11}{10} \quad (2)$$

choose $x=2$: $8 = -5A+5B+2C+D \Rightarrow 5A-2C-D = 5B-8 = \frac{27}{2}-8 = \frac{11}{2}$. (3)

From (1): $A = 3D + \frac{9}{10}$.

Then (2) becomes: $-C-D = \frac{11}{10} - A = \frac{11}{10} - \left(3D+\frac{9}{10}\right) = \frac{2}{10} - 3D = \frac{1}{5} - 3D$

and (3) becomes $-2C-D = \frac{11}{2} - 5A = \frac{11}{2} - 5\left(3D+\frac{9}{10}\right) = 1 - 15D$

We now need to solve $\left(-C+2D = \frac{1}{5}\right)\cdot(-2)$

$$\underline{-2C+14D = 1}$$

$$10D = \frac{3}{5} \Rightarrow D = \frac{3}{50}$$

Then $A = 3D + \frac{9}{10} = \frac{9}{50} + \frac{9}{10} = \frac{54}{50} = \frac{27}{25}$

and $C = 2D - \frac{1}{5} = \frac{6}{50} - \frac{1}{50} = \frac{4}{50}$

$$\therefore \frac{x^3}{(x-3)^2(1+x^2)} = \frac{27}{25}\frac{1}{x-3} + \frac{27}{10}\frac{1}{(x-3)^2} + \frac{-4x+3}{50(1+x^2)}.$$