MATH 142 (Section 502)
Prof. Meade

University of South Carolina
Fall 2008

Exam 4
November 25, 2008

Name: ______________________
Section 502

Instructions:

1. There are a total of 5 problems on 7 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	24	
2	12	
3	26	
4	18	
5	20	
Total	100	

Happy Thanksgiving!

1. (24 points) [8 points each] Determine if each series converges absolutely, converges conditionally, or diverges.

   (a) $\displaystyle\sum_{k=1}^{\infty}(-1)^k\frac{k^3}{e^k}$

   (b) $\displaystyle\sum_{k=1}^{\infty}\sin\left(\frac{k\pi}{2}\right)$

   (c) $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+1}{k^2+1}$

2. (12 points) Consider the convergent series

$$\frac{1}{1\cdot 2} - \frac{1}{2\cdot 2^2} + \frac{1}{3\cdot 2^3} - \frac{1}{4\cdot 2^4} + \cdots.$$

(a) Write this series in summation notation.

(b) How many terms are needed to approximate the sum of this series to two decimal place accuracy? (Do not approximate the sum!)

HINT: $0.5 = 1/2$, $0.05 = 1/20$, $0.005 = 1/200$, $0.0005 = 1/2000$, etc.

3. (26 points) [6 points each] Let $f(x) = \cos(2x)$.

   (a) Find the Taylor polynomials of orders $n = 0$, 1, 2, 3, and 4 about $x = \pi/2$.

EXTRA CREDIT Find the Taylor series for $f(x)$ about $x = \pi/2$.

4. (18 points) [6 points each] Find the radius of convergence for each of these power series.

(a) $\sum_{k=1}^{\infty} \frac{x^k}{k}$

(b) $\sum_{k=1}^{\infty} \frac{(3x)^k}{k!}$

(c) $\sum_{k=1}^{\infty} \left(\frac{4}{3}\right)^k (x+5)^k$

5. (20 points) [5 points each] Consider the Maclaurin series

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}x^k}{k}$$

(a) Show that the interval of convergence for this series is $-1 < x \leq 1$.

(b) A Maclaurin series for $\ln(1-x)$ that is valid for $-1 \leq x < 1$ is

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots. = -\sum_{k=1}^{\infty} \frac{x^k}{k}$$

Explain how this series, <u>and</u> its interval of convergence, are obtained directly from the Maclaurin series for $\ln(1+x)$.

(c) Use the given Maclaurin series for $\ln(1+x)$ and $\ln(1-x)$ to obtain the following Maclaurin series for $\ln\left(\frac{1+x}{1-x}\right)$:

$$\ln\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right). = 2\sum_{k=1}^{\infty} \frac{x^{2k-1}}{2k-1}$$

EXTRA CREDIT: How do you know this series converges for all $-1 < x < 1$?

(d) Use one of the series in this problem to find a series that converges to $\ln(1.5)$.
EXTRA CREDIT: How do you know the series converges to $\ln(1.5)$?