MATH 141 (Section 5 & 6)
Prof. Meade

Quiz 8
October 24, 2013

University of South Carolina
Fall 2013

Name: Key
Section: 005 / 006 (circle one)

1. (6 points) Find the absolute extreme values of the function $f(x) = (x^2 + 4x)^5$ on the interval $[-4, 1]$. Note: $2^5 = 32$, $3^5 = 243$, $4^5 = 1024$ and $5^5 = 3125$.

$f'(x) = 5(x^2+4x)^4(2x+4) = 10(x^2+4x)^4(x+2) = 0 \iff x^2+4x=0$ or $x+2=0$

$\iff x(x+4)=0$ or $x+2=0$

$\iff x=0, x=-2, x=-4$

x	$f(x)$
0	0
-2	$(-4)^5 = -1024$
-4	0
1	$5^5 = 3125$

abs. max: 3125 @ $x=1$

abs. min: -1024 @ $x=-2$

2. (4 points) Verify that the function $f(x) = \frac{x}{x+2}$ satisfies the hypotheses of the Mean Value Theorem on $[1, 4]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem. Note: $\sqrt{18} = 3\sqrt{2} \approx 4.2$.

Hypotheses of MVT: $f(x)$ continuous on $[1,4]$
$f(x)$ differentiable on $(1,4)$

$f'(x) = \frac{(x+2)(1) - 1\cdot x}{(x+2)^2} = \frac{2}{(x+2)^2}$

$m_{sec} = \frac{f(4)-f(1)}{4-1} = \frac{\frac{4}{6}-\frac{1}{3}}{3} = \frac{\frac{2}{3}-\frac{1}{3}}{3} = \frac{1}{9}$

solve: $f'(c) = \frac{1}{9}$

$\frac{2}{(c+2)^2} = \frac{1}{9}$

$(c+2)^2 = 18$

$c+2 = \pm\sqrt{18} = \pm 3\sqrt{2} \approx \pm 4.2$

$c = -2 \pm 3\sqrt{2} \approx -2 \pm 4.2$
$= 2.2$
-6.2

$\therefore\ c = -2+3\sqrt{2}$
≈ 2.2

Extra Credit (2 points) Give an example of a closed bounded interval $[a, b]$ on which the function $f(x) = \frac{x}{x+2}$ does not satisfy the conclusion of the Mean Value Theorem. Explain what hypothesis is not satisfied on your interval.

Any interval that includes $x=-2$.