MATH 141 (Section 3 & 4)
Prof. Meade

Exam 2
September 28, 2009

University of South Carolina
Fall 2009

Name: Key
Section: 003 / 004 (circle one)

Instructions:

1. There are a total of 8 problems (including the Extra Credit problem) on 7 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	20	
2	9	
3	12	
4	9	
5	10	
6	16	
7	12	
8	12	
Extra Credit	5	
Total	100	

Good Luck!

1. (20 points) Find each limit, or explain why the limit does not exist.

(a) $\lim_{x\to 1} e^{\cos(\pi x)} = e^{\cos(\pi \cdot 1)}$

$= e^{\cos(\pi)}$

$= e^{-1}$

(b) $\lim_{h\to 0^+} \frac{(h-1)^2+1}{h} = \lim_{h\to 0^+} \frac{h^2-2h+1+1}{h}$

$= \lim_{h\to 0^+} \frac{h^2-2h+2}{h}$

$= \lim_{h\to 0^+} \left(h - 2 + \frac{2}{h}\right)$

$= 0 - 2 + \infty = +\infty$

(c) $\lim_{t\to 2} \frac{t^2-4}{t^3-8} = \lim_{t\to 2} \frac{(t-2)(t+2)}{(t-2)(t^2+2t+4)}$

$= \lim_{t\to 2} \frac{(t+2)}{t^2+2t+4}$

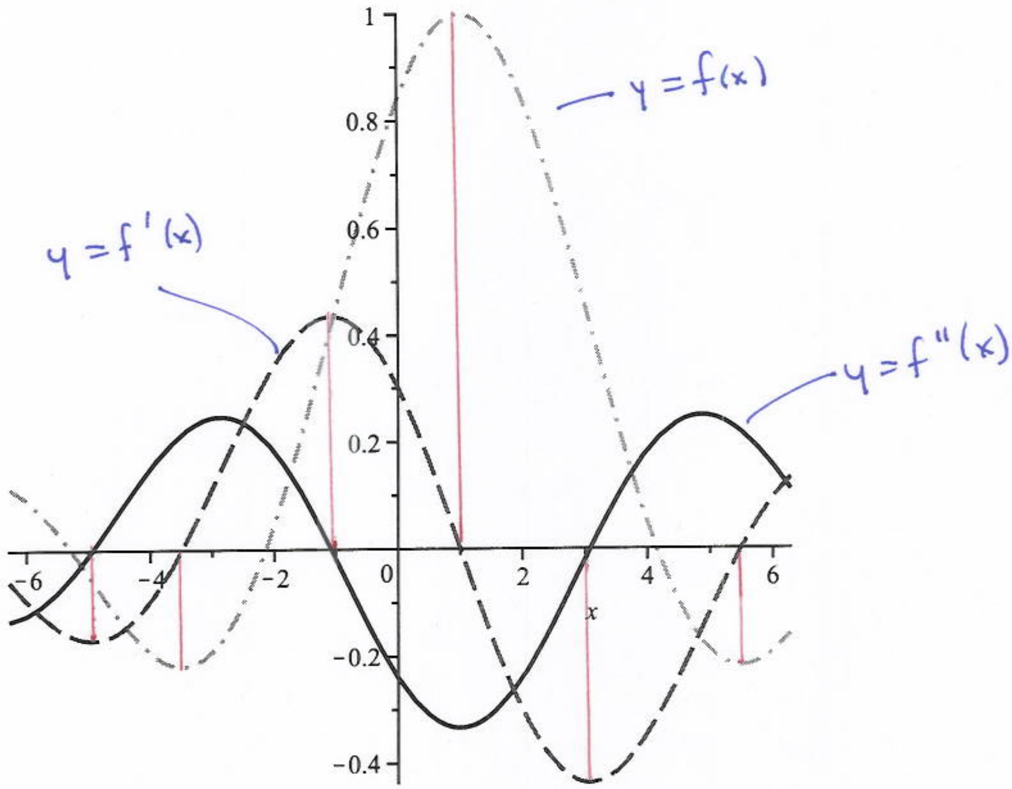
$= \frac{2+2}{4+4+4} = \frac{4}{12} = \frac{1}{3}.$

(d) $\lim_{t\to\infty} \frac{t^2+8}{\sqrt{t^3+4t+2}} = \lim_{t\to\infty} \frac{t^2(1+8/t^2)}{t^{3/2}\sqrt{1+4/t^2+2/t^3}}$

$= \lim_{t\to\infty} \underbrace{t^{1/2}}_{\to+\infty} \cdot \underbrace{\frac{1+8/t^2}{\sqrt{1+4/t^2+2/t^3}}}_{\to 1}$

$= +\infty.$

2. (9 points) The figure below shows the graphs of f, f', and f''. Identify each curve.



3. (12 points) Find the horizontal and vertical asymptotes of $y = \dfrac{2x^2+x-1}{x^2+x-2}$.

Horizontal asymptotes are found by looking at $\lim\limits_{x\to+\infty} y$ & $\lim\limits_{x\to-\infty} y$

$$\lim_{x\to+\infty} y = \lim_{x\to+\infty} \frac{2x^2+x-1}{x^2+x-2} = \lim_{x\to\infty} \frac{x^2(2+1/x-1/x^2)}{x^2(1+1/x-2/x^2)} = \lim_{x\to\infty} \frac{2+1/x-1/x^2}{1+1/x-2/x^2} = 2$$

$$\lim_{x\to-\infty} y = \lim_{x\to-\infty} \frac{2x^2+x-1}{x^2+x-2} = \lim_{x\to-\infty} \frac{2+1/x-1/x^2}{1+1/x-2/x^2} = 2$$

so there is only one horizontal asymptote: $y = 2$

Vertical asymptotes can occur when $\lim\limits_{x\to a} y = \pm\infty$.

Look at the denominator's zeros: $x^2+x-2 = (x+2)(x-1) = 0$

$$\lim_{x\to-2^+} \frac{2x^2+x-1 \to 1}{\underbrace{(x+2)}_{\to 0^+}\underbrace{(x-1)}_{\to -3}} = -\infty \quad \text{and} \quad \lim_{x\to 1^+} \frac{2x^2+x-1 \to 2}{\underbrace{(x+2)}_{\to 3}\underbrace{(x-1)}_{\to 0^+}} = +\infty$$

There are 2 vertical asymptotes: $x = -2$ and $x = 1$

4. (9 points) Find the values of a and b that make the function f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x + b & \text{if } x \ge 3 \end{cases}$$

Continuous at $x = 2$ when: $\lim_{x\to 2^-} f(x) = f(2)$: $f(2) = a(2)^2 - b(2) + 3 = 4a - 2b + 3$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x^2-4}{x-2} = \lim_{x\to 2^-} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2^-} (x+2) = 4$

so $4a - 2b + 3 = 4$ or $4a - 2b = 1$

Continuous at $x = 3$ when $\lim_{x\to 3^-} f(x) = f(3)$: $f(3) = 2(3) + b = 6 + b$.

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (ax^2 - bx + 3) = 9a - 3b + 3$

so $9a - 3b + 3 = 6 + b$ or $9a - 4b = 3$

Solving for a & b: $(4a - 2b = 1)(-2)$ then $4(1) - 2b = 1$

$9a - 4b = 3$ $\quad -2b = 1 - 4 = -3$

$a = 1$ $\quad b = \frac{-3}{-2} = \frac{3}{2}$

$\boxed{a = 1,\ b = 3/2}$

5. (10 points) Use the definition of the derivative to find the derivative of $f(x) = \sqrt{x+2}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \cdot \frac{(\sqrt{x+h+2} + \sqrt{x+2})}{(\sqrt{x+h+2} + \sqrt{x+2})}$$

$$= \lim_{h\to 0} \frac{(x+h+2) - (x+2)}{h(\sqrt{x+h+2} + \sqrt{x+2})}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h+2} + \sqrt{x+2})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h+2} + \sqrt{x+2}}$$

$$= \frac{1}{\sqrt{x+2} + \sqrt{x+2}} = \frac{1}{2\sqrt{x+2}}$$

6. (16 points) Differentiate each function.

(a) $F(x) = x^{-2/5}$

$F'(x) = -\frac{2}{5} x^{-2/5-1} = -\frac{2}{5} x^{-7/5}$

(b) $y = 5e^x + 3x^2 - 4$

$\frac{dy}{dx} = 5e^x + 6x$

(c) $B(u) = \frac{c}{u^6} = c u^{-6}$

$B'(u) = c(-6u^{-7})$

$= -6c u^{-7}$

(d) $g(t) = \frac{t^2 - 2\sqrt{t}}{t} = \frac{t^2 - 2t^{1/2}}{t} = t - 2t^{-1/2}$

$g'(t) = 1 - 2(-\frac{1}{2} t^{-3/2})$

$= 1 + t^{-3/2}$

7. (12 points) The equation of motion of a particle is $s = t^3 - 3t$, where s is in meters and t is in seconds ($t > 0$). Find

(a) the velocity as a function of t

$v = s' = 3t^2 - 3$

(b) the acceleration as a function of t

$a = v' = 6t$

(c) the acceleration after 2 s, and

$a(2) = 12$ m/sec^2

(d) the acceleration when the velocity is 0.

$v = 0$ when $3t^2 - 3 = 0$

$t^2 - 1 = 0$

$t^2 = 1$

$t = \pm 1$ (but consider only $t = 1 > 0$.)

$a(1) = 6$ m/sec^2

8. (12 points) Find an equation of the tangent line to the curve $y = x - \sqrt{x}$ at the point $(4, 2)$.

$= x - x^{1/2}$

A point on this line is $(4,2)$

The slope of the line is $m = \left.\frac{dy}{dx}\right|_{x=4}$:

$\frac{dy}{dx} = 1 - \frac{1}{2}x^{-1/2} = 1 - \frac{1}{2\sqrt{x}}$ so $m = \left.\frac{dy}{dx}\right|_{x=4} = 1 - \frac{1}{2\sqrt{4}} = 1 - \frac{1}{2\cdot 2} = \frac{3}{4}$.

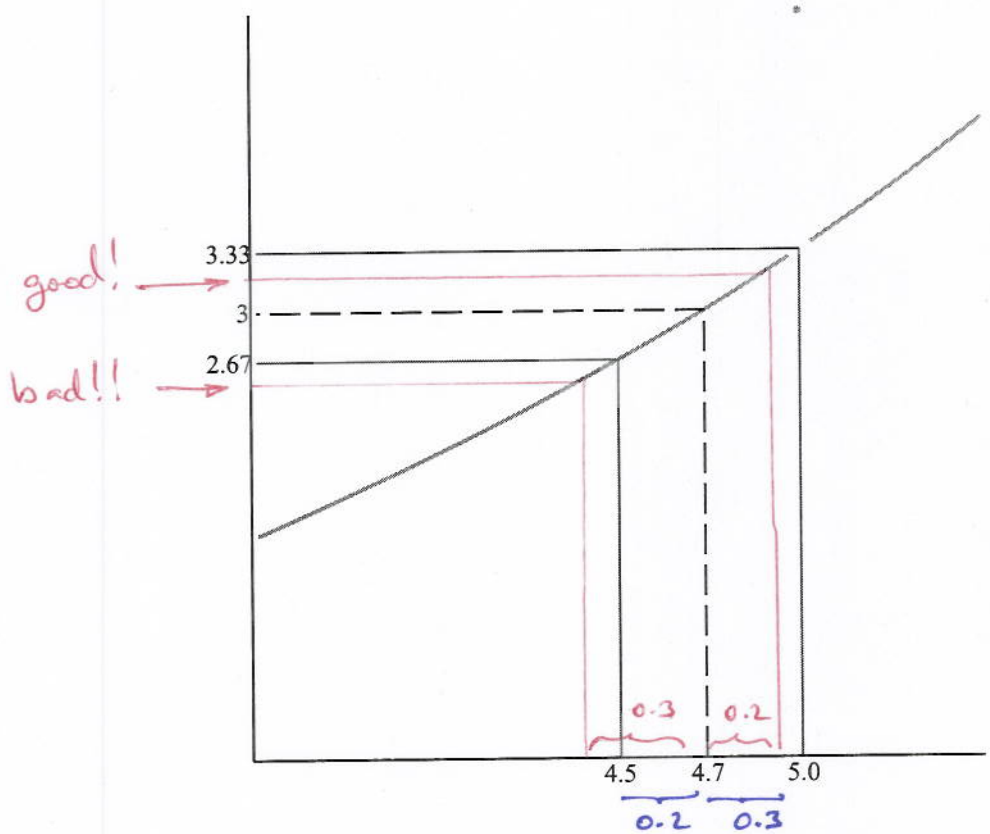
The equation of the tangent line is: $y - 2 = \frac{3}{4}(x-4)$

$y - 2 = \frac{3}{4}x - 3$

$y = \frac{3}{4}x - 1$

Extra Credit (5 points)Use the given graph of f to find a number δ such that

$$\text{if } 0 < |x - 4.7| < \delta \text{ then } |f(x) - 3| < 0.33$$



Choosing $\delta = 0.2$ will ensure that the function values stay between 2.67 & 3.33.