Math 141 (Section 11 \& 12)
Prof. Meade
Exam 4 - Practice
November 19, 2007

University of South Carolina
Fall 2007
Name: $\qquad$
Section: 011 / 012 (circle one)

Instructions:

1. There are a total of 7 problems on 6 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 20 |  |
| 2 | 15 |  |
| 3 | 15 |  |
| 4 | 15 |  |
| 5 | 15 |  |
| 6 | 10 |  |
| 7 | 10 |  |
| Total | 100 |  |

Study Smart!

1. (20 points) Let $f(x)=x e^{-x / 2}$. Find
(a) the interval(s) on which $f$ is increasing
(b) the interval(s) on which $f$ is decreasing
(c) the open intervals on which $f$ is concave up
(d) the open interval(s) on which $f$ is concave down
(e) the $x$-coordinates of all inflection points

Note: Be sure to show your work and to label your answers clearly.
2. (15 points) Let $f^{\prime}(x)=\frac{2-3 x}{\sqrt[3]{x+2}}$ be the first derivative of a continuous function $f$. Find all critical points of $f$ and determine whether each is a relative maximum, relative minimum, or neither.

Note: Show enough work to justify your answers.
3. (15 points) Sketch the graph of a continuous curve $y=f(x)$ with the following properties: $f(2)=4, f^{\prime}(2)=0, f^{\prime \prime}(x)>0$ for $x<2, f^{\prime \prime}(x)>0$ for $x>2$.
4. (15 points) Find the absolute maximum and absolute minimum values of $f(x)=\sin (x)-\cos (x)$ on $[0, \pi]$.
5. (15 points) Consider the following applied optimization problem:

A closed rectangular container with a square base is to have a volume of $2000 \mathrm{~cm}^{3}$. It costs twice as much per square centimeter for the top and bottom as it does for the sides. Find the dimensions of the container of least cost.

## Find

(a) the function to be maximized or minimized (indicate which it is)
(b) the interval of possible values for the variable in this problem

Do not solve the optimization problem.
6. (10 points) Determine if the hypotheses of the Mean-Value Theorem are satisfied for $f(x)=$ $\frac{1}{x-1}$ on the interval $[2,5]$. If they are, find all values of $c$ in this interval that satisfy the conclusion of the theorem.

7. (10 points) The position function of a particle moving on a horizontal $x$-axis is shown below.

(a) Is the particle moving left or right at time $a$ ?
(b) Is the acceleration positive, negative, or zero at time $a$ ?
(c) Is the particle speeding up or slowing down at time $a$ ?
(d) Is the particle speeding up or slowing down at time $b$ ?

