Example : #26 (p. 298)

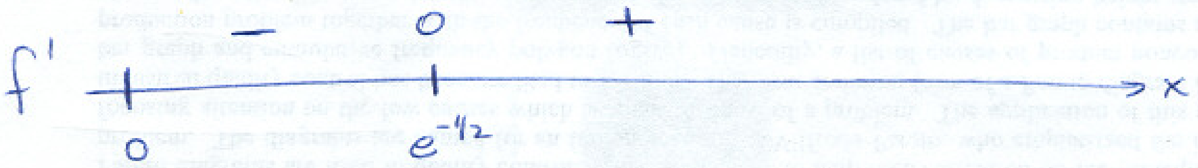
$$f(x) = x^2 \ln x, \quad x > 0.$$

$$f'(x) = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x = x(2\ln x + 1)$$

Because $x > 0$, we always have $x > 0$.

Look at $2\ln x + 1$: $2\ln x + 1 = 0 \iff \ln x = -1/2$

$\iff x = e^{-1/2} \approx 0.61$



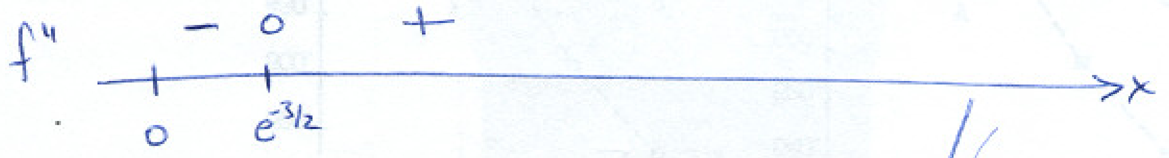
$$\left\{\begin{array}{l} 0 < x < e^{-1/2}: \ 2\ln x + 1 < 0 \quad \text{so} \quad x(2\ln x+1) < 0, \text{ i.e. } f'(x) < 0 \quad (f \text{ decreasing}) \\ e^{-1/2} < x < \infty: \ 2\ln x + 1 > 0 \quad \text{so} \quad x(2\ln x+1) > 0, \text{ i.e. } f'(x) > 0 \quad (f \text{ increasing}) \end{array}\right\}$$

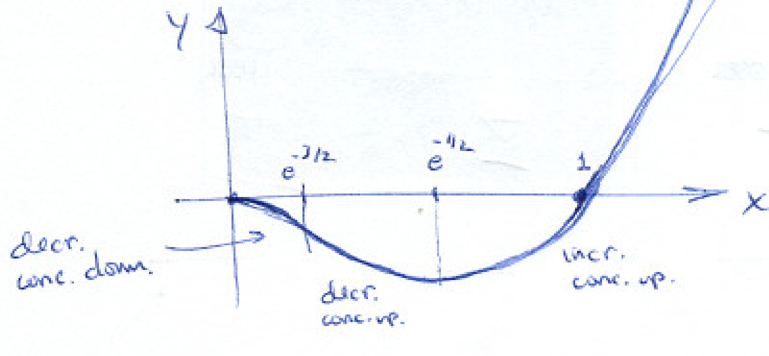
$$f''(x) = 2\ln x + 2x \cdot \frac{1}{x} + 1 = 2\ln x + 3$$

$$2\ln x + 3 = 0 \iff \ln x = -3/2 \iff x = e^{-3/2} \approx 0.22$$ ← inflection point

$0 < x < e^{-3/2}$: $2\ln x + 3 < 0$ so $f''(x) < 0$ (f concave down)

$e^{-3/2} < x < \infty$: $2\ln x + 3 > 0$ so $f''(x) > 0$ (f concave up).





* For a better plot, see the Curve Analysis tutor