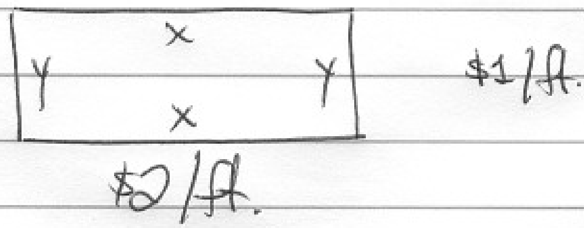
p. 350
#11



$$A = \boxed{3200 = xy} \implies y = \frac{3200}{x}$$

$$C = 2x+y+2x+y = 4x+2y = 4x + 2\left(\frac{3200}{x}\right)$$
$$= 4x + \frac{6400}{x}$$

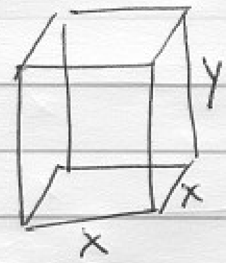
min cost.
s.t. $A = 3200$

$x > 0$
$y > 0$

min $C(x) = 4x + \frac{6400}{x}$
s.t. $x > 0$.

sol'n: $x = 40$
$y = 80$

p. 350
#20.



$$V = x^2 y = 2250 \Rightarrow y = \frac{2250}{x^2}$$

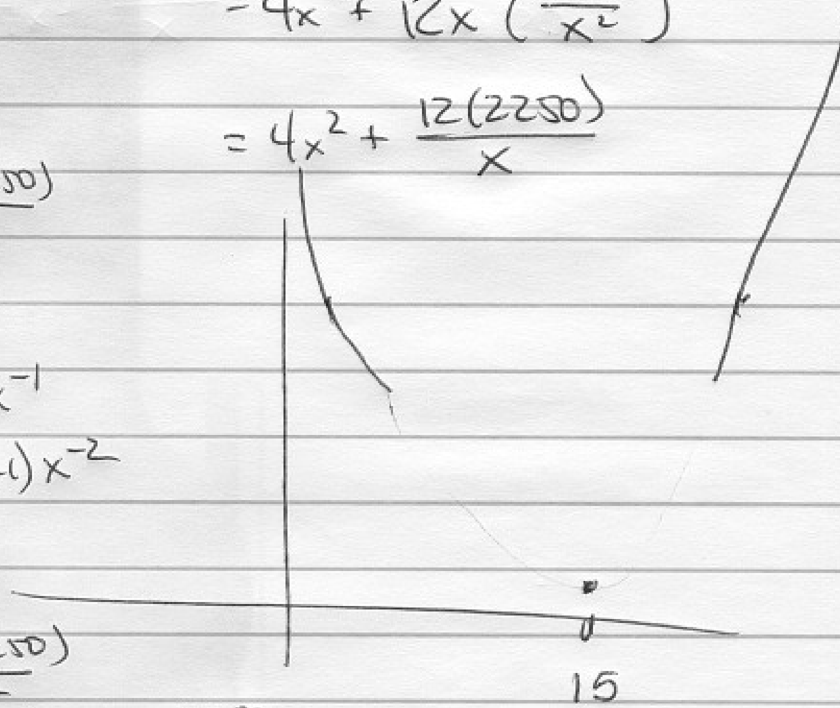
top/bottom: $\$2/in^2$.
sides : $\$3/in^2$.

$2 \cdot x^2 \cdot \$2/in^2 = 4x^2$
$4 \cdot xy \cdot \$3/in^2 = 12xy$

min (total cost)
s.t. $V = 2250$

$$C = 4x^2 + 12xy$$
$$= 4x^2 + 12x\left(\frac{2250}{x^2}\right)$$
$$= 4x^2 + \frac{12(2250)}{x}$$

min $C(x) = 4x^2 + \frac{12(2250)}{x}$
s.t. $x > 0$.



$$C(x) = 4x^2 + 12(2250)x^{-1}$$
$$C'(x) = 8x + 12(2250)(-1)x^{-2}$$
$$= 8x - \frac{12(2250)}{x^2}$$

$C'(x) = 0$: $8x = \frac{12(2250)}{x^2}$
$$x^3 = \frac{12(2250)}{8} \frac{3}{2} = \frac{6750}{2} = 3375$$
$$x = 15$$

because $\lim_{x\to\infty} C(x) = \infty$ and $\lim_{x\to 0^+} C(x) = \infty$

we know $x = 15$ is the min. cost: dimensions: $x = 15$ in.
$y = \frac{2250}{15} = 10$ in.

Riemann Sums

(p 409) $A = \lim_{n \to +\infty} \sum_{k=1}^{n} f(x_k^*) \Delta x$ (uniform subinterval

$= \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*) \Delta x_k.$

must be given a & b

$= \int_a^b f(x)\,dx$

Uniform subinterval: $\Delta x = h = \frac{b-a}{n}$

Ex: (Exam 4) $\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} \frac{\sin^2(x_k^*)}{1+x_k^*} \Delta x_k.$

$$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{\sin^2\left(k\frac{\pi}{n}\right)}{1+\frac{k\pi}{n}} \left(\frac{\pi}{n}\right)$$

Riemann Sum w/uniform subinterval:

$$\int_a^b \frac{\sin^2(x)}{1+x}\,dx$$

where $x_k = k\frac{\pi}{n} = k\Delta x$ $\quad a = x_0 = 0 \cdot \Delta x = 0.$

$\Delta x = \frac{\pi}{n}$ $\quad b = x_n = n \cdot \Delta x = n\left(\frac{\pi}{n}\right) = \pi$

so $\int_0^{\pi} \frac{\sin^2(x)}{1+x}\,dx$

Exam 4
#6c

$$\int_{-1}^{1} 3+2\sqrt{1+x^2}\,dx = \int_{-1}^{1} 3\,dx + \int_{-1}^{1} 2\sqrt{1\cancel{+}x^2}\,dx$$

$$= 3x\Big|_{-1}^{1} + \int_{-1}^{1} 2\sqrt{1\cancel{+}x^2}\,dx$$

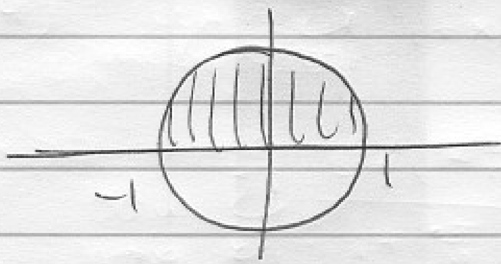
$$= 3-(-3) + \int_{-1}^{1} 2\sqrt{1\cancel{+}x^2}\,dx$$

$$= 6 + \int_{-1}^{1} 2\sqrt{1\cancel{+}x^2}\,dx$$

* Substitution will not work for this integral
** cannot use the FTC.

$$= 6+2\int_{-1}^{1} \sqrt{1-x^2}\,dx$$

$y=\sqrt{1-x^2}$ $\quad (y^2=1-x^2,\ y^2+x^2=1)$



$$\int_{-1}^{1} \sqrt{1-x^2}\,dx$$

$$= \frac{1}{2}\left(\pi (1)^2\right)$$

$$= \frac{1}{2}\pi.$$

so

$$\int_{-1}^{1} 3+2\sqrt{1-x^2}\,dx = 6 + 2\left(\tfrac{1}{2}\pi\right)$$

$$= 6+\pi.$$

Exam 2 #6b.

$$\lim_{\Delta x\to 0} \frac{\sqrt{1+\Delta x}-1}{\Delta x}$$

Key: $$f'(a) = \lim_{\Delta x\to 0} \frac{f(a+\Delta x)-f(a)}{\Delta x}$$

$$= \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

Here: to make this work we need

$$f(a+\Delta x) = \sqrt{1+\Delta x}$$

$$f(a) = 1 = \sqrt{1}$$

this suggests: $a=1$, $f(x)=\sqrt{x} = x^{1/2}$

then $f'(x) = \frac{1}{2}x^{-1/2}$.

$$f'(1) = \frac{1}{2}(1)^{-1/2} = \frac{1}{2}.$$

so $$\lim_{\Delta x\to 0} \frac{\sqrt{1+\Delta x}-1}{\Delta x} = \frac{1}{2}.$$

(or, use l'Hopital's Rule)

§3.2 (p.188)

#18. $y = x^2 - x$

Find $\frac{dy}{dx}$ by the definition.

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

~~$f(x+h) =$~~

$$f(x) = x^2 - x$$

$$f(x+h) = (x+h)^2 - (x+h)$$
$$= x^2 + 2hx + h^2 - x - h$$

now,

$$f(x+h) - f(x) = (\cancel{x^2} + 2hx + h^2 - \cancel{x} - h) - (\cancel{x^2} - \cancel{x})$$
$$= 2hx + h^2 - h$$

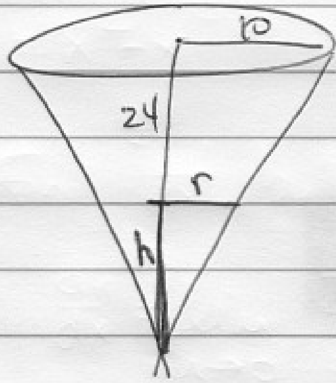
$$\frac{f(x+h) - f(x)}{h} = \frac{2hx + h^2 - h}{h} = 2x + h - 1$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 2x + h - 1 = 2x - 1$$

so $f(x) = 2x - 1$.

Related Rates (§3.7)

p. 224
#25



Know: $V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = 20 \text{ ft}^3/\text{min}.$

$h = 16$

Find: $\frac{dh}{dt}$ when $h = 16$ ft.

By similar triangles: $\frac{h}{24} = \frac{r}{10}$

so $r = \frac{10}{24} h = \frac{5}{12} h.$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{5}{12}h\right)^2 h$$

$$= \frac{25\pi}{432} h^3$$

$$V(t) = \frac{25\pi}{432} h(t)^3$$

differentiate w.r.t time: $\frac{dV}{dt} = \frac{25\pi}{432} 3h^2 \cdot \frac{dh}{dt}$

$$= \frac{25\pi}{144} h^2 \frac{dh}{dt}$$

plug in values:

$$20 = \frac{25\pi}{144} (16)^2 \frac{dh}{dt}$$

so $\frac{dh}{dt} = \frac{\overset{4}{\cancel{20}} \cdot \overset{9}{\cancel{144}}}{\underset{5}{\cancel{25}} \cdot \underset{4}{\cancel{16^2}} \cdot \pi} = \frac{9}{20\pi} \text{ ft/min}$