MATH 141 (Section 1 & 2)
Prof. Meade

University of South Carolina
Fall 2004

Exam 4 – Practice
November 23, 2004

Name: ____________________
Section: 001 002 (circle one)

Instructions:

1. There are a total of 7 problems on 7 pages. Check that your copy of the exam has all of the problems.
2. Calculators may not be used for any portion of this exam.
3. You must show all of your work to receive credit for a correct answer.
4. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.
5. **This exam is a little longer than what I expect you to be able to complete in a one hour class. But, it is good practice when studying for the exam.**

Problem	Points	Score
1	16	
2	20	
3	15	
4	9	
5	5	
6	15	
7	20	
Total	100	

Study Hard! Ask Questions!!

1. (20 points) Assume that the function f is continuous and the signs of the first and second derivatives are as shown in the following table:

Interval	Sign of $f'(x)$	Sign of $f''(x)$
$x < -1$	+	-
$-1 < x < 1$	-	-
$1 < x < 4$	-	+
$4 < x < 7$	-	-
$7 < x < 12$	-	+
$12 < x$	+	+

Find each of the following:

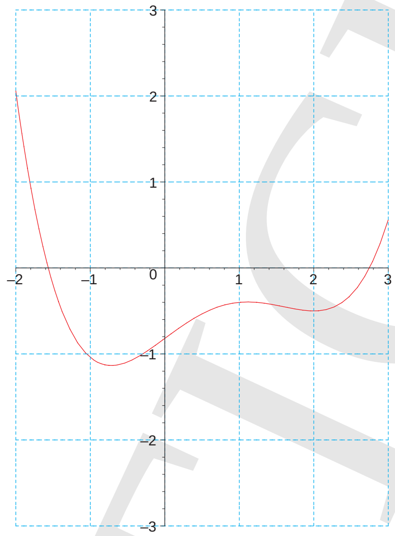
(a) the intervals on which f is increasing

(b) the intervals on which f is concave down

(c) the x-coordinates of all critical numbers

(d) the x-coordinates of all inflection points

2. (20 points) Use the following graph of $y = f'(x)$ to answer the following questions about the unknown function f.



Hint: *The numbers appearing in your answers must be taken from the following list:*
-2.00, -1.56, -0.96, -0.72, 1.12, 2.00, 2.75, 3.00

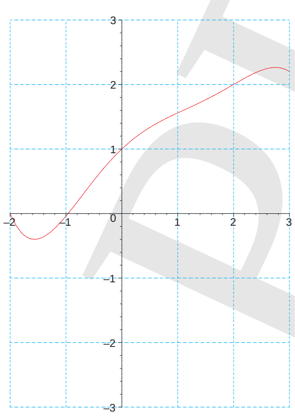
(a) On what interval(s) is f decreasing?

(b) Give the x-coordinate(s) of all points where f has a relative maximum.

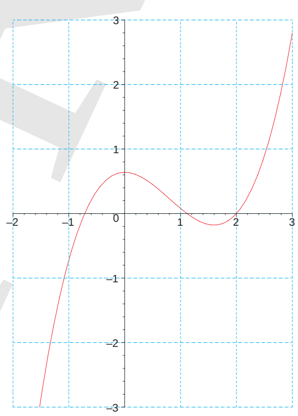
(c) On what interval(s) is f concave up?

(d) Which of the following graphs is the graph of $y = f(x)$? [Circle one: (a) (b) (c) (d)]

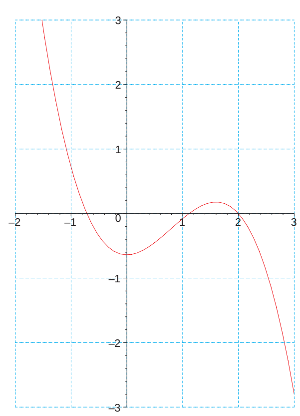
(e) Which of the following graphs is the graph of $y = f''(x)$? [Circle one: (a) (b) (c) (d)]



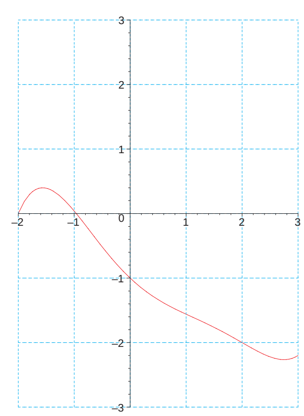
(a)



(b)



(c)



(d)

3. (15 points) Let $s(t) = 5t^2 - 22t$ be the position function of a particle moving along a coordinate line, where s is in feet and t is in seconds.

(a) After 1 second ($t = 1$), in what direction is the particle moving?

(b) Find the maximum velocity of the particle during the time interval $1 \leq t \leq 3$.

(c) When, during the time interval $1 \leq t \leq 3$, is the particle farthest from the origin?

Remember to provide a short explanation for each answer.

4. (10 points) Solve the problem:

   A closed rectangular container with a square base is to have a volume of 2000 cm^3. The material used for the top and bottom of the container costs twice as much per square centimeter as the material for the sides. Find the dimensions of the container with least cost.

5. (10 points) State Rolle's Theorem.

6. (10 points) Find the absolute maximum and absolute minimum values of $f(x) = \frac{\ln(2x)}{x}$ on the interval $[1, e]$.

Use $e \approx 2.7$ *and* $\ln(2e) \approx 1.7$.

7. (20 points)

(a) Express the limit

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} 4x_k^*(1 - 3x_k^*)\Delta x_k$$

as a definite integral with $a = -3$ as the lower limit of integration and $b = 3$ as the upper limit of integration.

Do not evaluate the integral!

(b) Sketch the region whose net signed area is represented by the definite integral

$$\int_{-2}^{4} |2x - 1| \; dx.$$

Do not evaluate the integral!

(c) Use properties of the definite integral and appropriate geometric formulas to evaluate

$$\int_{0}^{2} x + 2\sqrt{4 - x^2} \; dx.$$

(d) Suppose $\int_0^1 f(x) \; dx = -2$ and $\int_0^4 f(x) \; dx = 1$, find $\int_1^4 f(x) \; dx$.