An Introduction to Maple and Maplets for Calculus

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Just as Word has its documents and Excel has its spreadsheets, Maple has worksheets. [The underlined green text is a hyperlink. Click it and see what happens.] Worksheets can be identified by their .mw extension.

Worksheets can contain text, input, output, and graphics (2d or 3d, including animation). [Help pages contain a set of examples; this can be a very useful way to learn what a command does.] The text can include images, bulleted or numbered lists, tables, hyperlinks, and many other features.

▼ Maple as a Calculator

Maple input regions are identified by a prompt (the red greater than sign). All Maple commands must end with a `;` or a `:` . To execute a command, just press Enter.

```
> 3+4*5;
> 10!;
```

You can enter commands by typing. Many operations can also be entered using the palettes (along the left-hand column of the Maple window). See if you can compute $\ln(100)$ and $\log_{10}(100)$.

Notice that Maple returns exact results. If you want an approximation, you have to ask for it. The easiest way to request an approximation is simply to include a decimal point in the input.

To understand the next examples, you need to understand that Maple is case sensitive: **pi**, **Pi**, and **PI** refer to three different objects. **pi** and **PI** are the lower and upper case Greek letters; only **Pi** is the mathematical constant. (Notice how the next input region contains three lines of code, and that each line ends with a **;** . What happens when the first two **;** are replaced by **,** ? What if they are simply omitted? Try it!)

```
> pi;
> Pi;
> PI;
```

To emphasize how Maple treats these differently, move the cursor to the first π (the output, not the input) and hold down the right-hand mouse button. Maple will produce a context menu showing the most likely operations to perform with this object. Only one of these contains an entry called Approximate; click on this, then click on the number of digits you want to see in Maple's answer. (The result will appear in a new input region inserted immediately after the one where the object is defined - before this text. Try it! Then, open the next subsection.)

▼ Special Note about Floating-Point Calculations

If you followed the directions you now know the name of the Maple command that forces floating-point calculations. It is important to realize that the number of digits is the number of digits **used** in the calculation. This does not mean that all of these digits are correct. If the number of digits is omitted from the evalf command, the default number of digits is 10. This default can be changed by changing the value of the environment variable Digits.

```
> Digits;
> evalf( Pi );
> Digits := 20;
> evalf( Pi );
> Digits := 10;
```

▼ An even more special note

You now know how to make an assignment: use **:=** . The single character **=** is used to represent equality in an equation.

To test your understanding, try to explain the results of the following four commands. (Maple's trigonometric functions all assume the angles are given in radians.)

Which one does not evaluate to a number? Why?

```
> sin( pi/3 );
> sin( 3.14149/3 );
> sin( Pi/3 );
> sin( Pi/3. );
```

Feel free to modify any of these expressions, or to add more of your own. To make new input regions, click on the icon that looks like [> in the tool bar at the top of this window. (A description of the icon's function is displayed if you let the cursor hover over the icon without clicking anything.

▼ Maple as a Tool for Doing Mathematics

The next sections are all designed to help us move towards deciding which is larger, π^e or e^π. If you get done with this, or if you are already familiar with Maple, take a look at the sample maplet from Maplets for Calculus and the Questions at the end of this worksheet. Each one is more difficult than the previous one. Possible solutions are provided to give you some hints, if needed.

▼ Assignments

The two-character sequence **:=** is used to make an assignment. The single character **=** is used to construct an equation.

The exponential function is **exp**. You can type, or use the palettes, to fill in the right-hand sides of the following assignments that define the functions $f(x) = x^e$ and $g(x) = e^x$.

```
> f := ;
```
```
> g := ;
```

▼ Expressions vs. Functions

Notice that we did not write f(x) := ... . Technically, we have defined expressions.

In Maple, a function is a mapping from an input to an output (just as a function is actually defined in mathematics).

To define an actual function, the syntax is a little different. The two characters **->** are used to indicate the mapping:

```
> F := x -> x^exp(1);
```
The distinction between expressions and functions becomes clearer when you have to want to evaluate it at a specific input.
```
> F(2);
```
```
> f(2);
```
```
> eval( f, x=2 );
```
Most new users find it easier to work with expressions. There is an entry in the Expressions palette that allows us to not remember the exact syntax of the eval command.

▼ Plotting

You should load the plots package prior to doing any plotting:
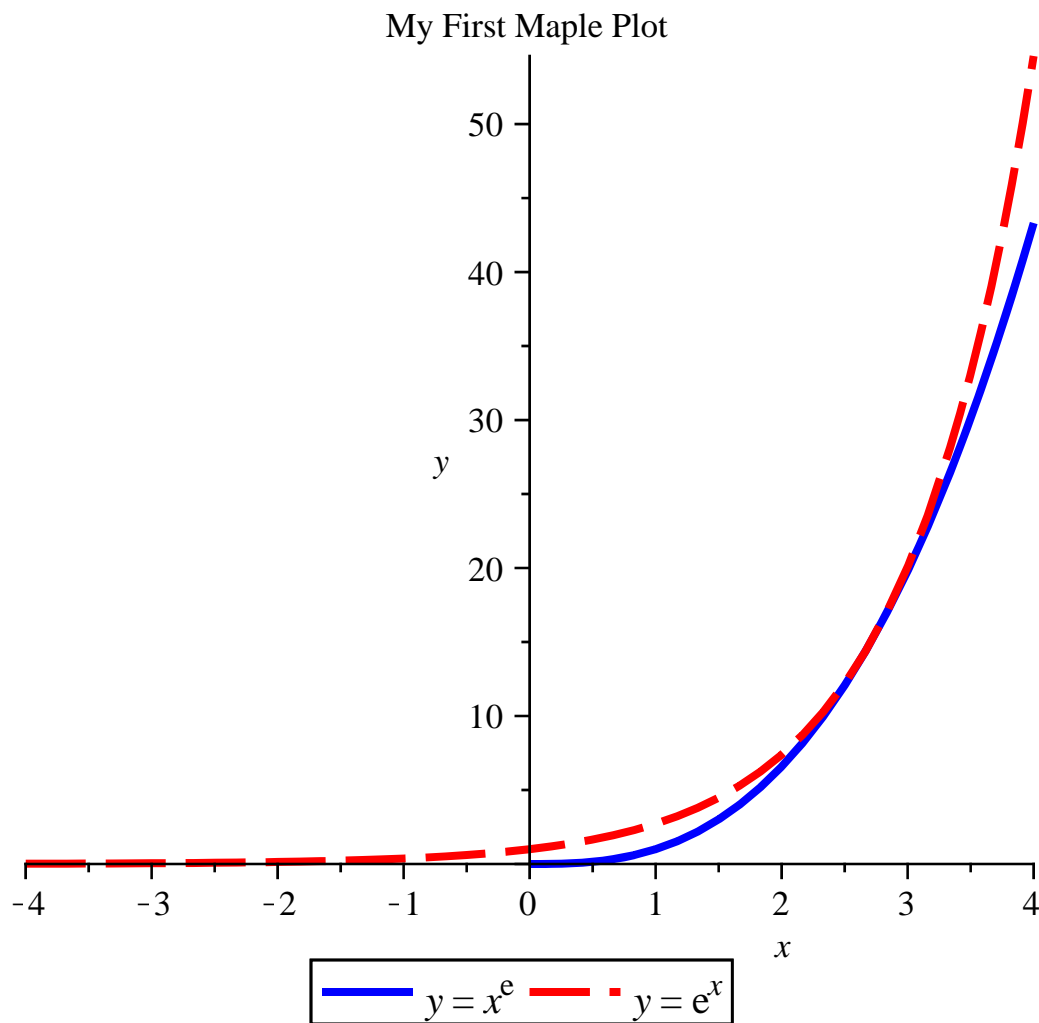
```
> with( plots );
```

The basic ingredients of a Maple plot are the expression to be plotted and the domain on which it should be plotted.

```
> plot( f, x=-4..4 );
```

The interface allows you to make additions and changes to this plot. For example, if you copy the exp(x) from the output of the definition of g and paste it onto the plot, both functions will be displayed. To make changes to a specific curve, position the cursor over a curve and click; then call up the context menu. Use this to change the color, style, etc. of the two curves. To change general properties of the graph, move the cursor away from all curves and call up the context menu.

▼ Challenge [plot options]

Try to make your plot look like this. You will need to change the colors, and line styles, add a title, a label on the y-axis, and a legend. All of this

<p style="text-align:center">My First Maple Plot</p>

Legend: $y = x^e$ $\quad$ $y = e^x$

▼ *Solution*

This plot can also be created directly by adding optional argument to the plot command.

```
> plot( [f,g], x=-4..4,
```

```
            color=[blue,red], thickness=3, linestyle=[solid,
        dash],
            labels=[x,y], legend=[y=f,y=g], title="My First
        Maple Plot" );
```

In 2D, Maple can also work with implicitly defined functions, parametric curves. In 3D you can work with curves or surfaces given in a variety of different ways. Animation are easily created in either 2D or 3D.

Solving Equations (and Inequalities)

Challenge [classic inequality]

Which is larger: π^e or e^π ?

Hint:

To get started, evaluate the two expressions when $x = \pi$:

```
> eval( [f,g], x=Pi );
```

Use what you learned earlier to find the floating-point approximations to these two numbers.

Referring to Previous Results

How can I refer to an unassigned previous result?
1. revise the previous command so that it is assigned to a name

Example

```
> q := eval( [f,g], x=Pi );
```

2. use the label along the right-hand side of the window

Example

Here, we use the label (option 2):

```
> evalf( ?? );
```

Before moving on, position the cursor inside the label in the above command and click the right-hand mouse button. The last entry in the context menu is Label Reference; click it. Notice that the expression referred to by this reference now appears in the command. Pretty neat, I think. (While label references will be automatically updated as you modify a worksheet, they do break when the quantity is deleted from the workhseet.)

3. use the ditto operator (%)

Example

```
> eval( [f,g], x=Pi );
```
```
> evalf( % );
```

Note that this approach can be unstable. It depends upon the output of the previous result, independent of whether that was above or below this location, or even if it's been deleted

from the worksheet. Use this only when you are certain it won't cause a problem.

This classic example, and the plot, might give us the idea that exponentials are always at least as big as powers.

To find intersections of the two curves we are studying requires setting **f** and **g** equal to each other and solving for **x**:

```
> solve( f=g, x );
```

▼ Note

Note that the **x** can be omitted from this command only because there is only one indeterminate in this equation.

Maple can also solve systems of equations for more than one variable; in these cases the list of of equations and the list of variables are usually given as a list, with square brackets, or a set, with squiggly brackets.

This is consistent with what we saw in the plot, but it does not answer the question. For that we need to solve an inequality:

```
> solve( f<g, x );
```

Maple's output is telling us that the inequality is true for all values of x except $x = e$.

▼ Challenge [meaning of no output]

Explain Maple's response to this command:

```
> solve( f>g, x );
```

▼ *Solution*

Maple gives us no answer. This means this inequality is not satisfied for any value of x. If Maple was unable to solve the equation, it's response would be different. For example:

```
> solve( cos(x)=x, x );
> solve( cos(x)>x, x );
```

▼ Challenge [assumptions]

Why can't Maple evaluate this limit?

```
> limit( exp(a*x), x=infinity );
```

▼ *Hint*

To learn how to tell Maple about assumptions that you know about a parameter, see the online help for assuming .

This concludes our basic introduction to Maple.

Before I end I want to give a little advertisement for my 50-minute sesion on Sunday at 16:00 in Room 213 in the Math Building.

▼ Maplets for Calculus

Maplets for Calculus is a collection of more than 100 maplets written by Phil Yasskin at Texas A&M University and me. They are intended to help students develop basic skills on a wide variety of calculus, and pre-calculus, topics. The one that is most relevant to the general themes in this workshop is Computing Tangent Lines.

(Click on this hyperlink, and be patient while Java starts. Once this starts, click on New Problem, and fill in the boxes. You can check your answers at each step by clicking on the Check button; the Show button will fill in the blank with the correct answer. Notice that answers can be entered in many different forms - so long as Maple can simplify your expression to one that agrees with the correct answer.)

I will be giving a full demonstration of the Maplets for Calculus package in .

▼ Question 1 [Comparison of Exponential and Power Functions]

For what values of x is it true that $x^2 < 2^x$?

▼ Hint

While a plot is not a proof, a good plot can certainly help get you started or to confirm the reasonableness of the analytic solution.

▼ Workspace

Use this space to work out your solution to this problem. (Remember how you can add more input regions, if needed.)

▼ Solution

```
> plot( [x^2,2^x], x=-2..5 );
> solve( x^2=2^x, x );
> -2*LambertW((1/2)*ln(2))/ln(2);
```

```
> evalf[5]( ?? );
> solve( x^2<2^x, x );
> evalf( [??] )[];
```

▼ Question 2 [Tangent Lines]

Find all points on the graph of y=x^3 whose tangent line passes through the point (0,10)? the point (1,10)? the point (2,5)?

▼ Hint

While a plot is not a proof, a good plot can certainly help get you started or to confirm the reasonableness of the analytic solution.

▼ Workspace

Use this space to work out your solution to this problem. (Remember how you can add more input regions, if needed.)

```
> 3
```

▼ Solution

▼ *for (0,10)*

```
> f := x^3;
> P1 := plot( f, x=-2..3 );
> P2 := plot( [ [0,10] ], style=point, color=green,
    symbolsize=18 );
> display( P1, P2 );

> df := diff( f, x );
> ytan := y = eval(f,x=x0)+eval(df,x=x0)*(x-x0);
> eqn := eval( ytan, [x=0,y=10] );
> S := [solve( eqn, x0 )];
> evalf( S );

> P3 := plot( eval( rhs(ytan), x0=S[1] ), x=-2..6, color=
```

```
      blue, linestyle=dashdot );
> display( P1, P2, P3, view=[-2..6,-20..50] );
```

▼ *for (1,10)*
```
> f := x^3;
> P1 := plot( f, x=-2..3 );
> P2 := plot( [ [1,10] ], style=point, color=green,
  symbolsize=18 );
> display( P1, P2 );

> df := diff( f, x );
> ytan := y = eval(f,x=x0)+eval(df,x=x0)*(x-x0);
> eqn := eval( ytan, [x=1,y=10] );
> S := [solve( eqn, x0 )];
> evalf( S );

> P3 := plot( eval( rhs(ytan), x0=S[1] ), x=-2..6, color=
  blue, linestyle=dashdot );
> display( P1, P2, P3, view=[-2..6,-20..50] );
```

▼ *for (3,10)*
```
> f := x^3;
> P1 := plot( f, x=-3..6, color=cyan );
> P2 := plot( [ [3,10] ], style=point, color=magenta,
  symbolsize=18 );
> display( P1, P2 );

> df := diff( f, x );
> ytan := y = eval(f,x=x0)+eval(df,x=x0)*(x-x0);
> eqn := eval( ytan, [x=3,y=10] );
> S := [solve( eqn, x0 )];
> evalf( S );

> L := [seq( evalf(eval( rhs(ytan), x0=Re(s) )), s=S )];
> P3 := plot( L, x=-3..6, linestyle=dashdot, legend=[seq(x
  [0]=Re(s),s=evalf[2](S))] );
> display( P1, P2, P3, axes=box, view=[-3..6,-20..150] );
```

Bonus

Are there any points $(a, 10)$ that are on exactly two tangent lines to the graph of $y = x^3$? Explain.

Question 3 [Shrinking Sphere]

Solve the Shrinking Sphere Problem.

Recall that this problem involves two circles: a fixed circle of radius a that is centered at $(a, 0)$ and a variable circle with radius r centered at the origin. Let P be the point $(0, r)$ and Q be the point of intersection of the two circles in the first quadrant. The point R is the point $(x_0, 0)$ on the line that passes through both P and Q and on the x-axis.

The question is what happens to x_0 as r decreases to 0?

Hint

The explicit formula for x_0 can be written as $2\,a + \sqrt{4\,a^2 - r^2}$. So, $\lim\limits_{r \to 0} x_0 = 4\,a$.

Workspace

Use this space to work out your solution to this problem. (Remember how you can add more input regions, if needed.)