Mathematical Modeling

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Overview

"Mathematical modeling" is used to describe many different types of problems. One common use is the process of developing equations to describe a real-world situation based on accepted properties of the system. An example is the use of Newton's Laws of Motion to develop equations for the trajectory of a rocket. Here, the form of the governing equation is given but the parameters in the equation are determined from observed data.

Maple Essentials

- The *Curve Fitting* maplet is started from the Maple 9.5 user interface under the Tools menu:

  Tools → Assistants → Curve Fitting ...

  The *Curve Fitting* maplet returns an expression to the Maple worksheet.
- To plot an expression that appears in a Maple worksheet, use the cursor to select what you want to plot, then click the right mouse button. Wait for a menu to appear. Select **Plots → Plot Builder**.
- To add another function to an existing Maple plot, use the cursor to select the expression to be plotted and click the right mouse button (as above). When the menu appears, select **Copy**. Position the cursor over the plot, press the right mouse button and select **Paste**. (It is also possible to drag-and-drop the expression onto the desired plot.)

Preparation

Read Examples 1 (p. 81) and 3 (p. 83) in §1.7 of Anton. Shifts of functions and properties of lines will be used in the project; review these topics as needed.

Activities

- Start a Maple session.
- Launch the *Curve Fitting* maplet.
- Work Exercises #6 – 8, and 19.
  Be sure you learn to use the *Curve Fitting* maplet to plot data and to find the best least squares fit to the data. (Be sure you understand #6(b) and how to work with a square-root model for #19.)
- Work on Project 1 (see below).
- If you are using a computer in a SAM lab, remember to logout.

Assignment

For Project 1, prepare a neat and complete project report for the problem on the back of this page.

Your TA will provide details about the format of your project and the due date.

An important problem addressed by calculus is that of finding a good linear approximation to the function $f(x)$ near a particular x-value, say $x = p$. One possible approach (not the best) is to sample values of the function near $x = p$, find the least squares line for this data, and translate the least squares line so that it passes through the point $(p, f(p))$, which is on the graph of $y = f(x)$. Let $f(x) = 2 + x + |\sin(x)|$.

1. Let $p = 2$.
   (a) Make a table of $(x, f(x))$ values for five (5) x equally-spaced values on the interval $[p-0.1, p+0.1]$. **Note:** *The function values in this table must be correct to at least three (3) digits to the right of the decimal point. You can round or truncate, but be consistent and be sure to mention this in your written report.*
   (b) Find the least squares line for the data in (a).
   (c) Find the equation of the line parallel to the least squares line found in (b) that passes through the point $(p, f(p))$.
   (d) Create a figure that displays the graph of $y = f(x)$ and the graph of the line found in (c).
   (e) Is the graph of the line found in (c) a good linear approximation to the graph of $y = f(x)$ near the point $(p, f(p))$?
2. Let $p = 0$.
   Repeat steps (a)–(e) from 1., above.
3. Give a general rule for the values of p for which this method yields a good approximation to the graph of $y = f(x)$ (near $x = p$). To get started on this, it might help to consider several more values of p, for example $p = 1$, $p = \frac{\pi}{3}$, $p = \frac{\pi}{2}$, $p = 3$, $p = \pi$, $p = \frac{3\pi}{2}$, $p = 2\pi$, $p = -\frac{\pi}{2}$, and $p = -\pi$.

   Note: *It is not necessary to summarize the results for each of these cases in your report. All you need to provide is the general rule and how you arrived at this conclusion.*

This problem is based on #41 on page 101 of Anton.