

A More Rigorous Approach to Limits

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Overview

The rigorous ϵ - δ definition of limits can be difficult for students to grasp. This lab is designed to provide visual and interactive tools for working with these concepts.

Maple Essentials

- The *EpsilonDelta* maplet is available from USC at the URL:

<http://www.math.sc.edu/~meade/141L-F04/maplets/CalcUSC/EpsilonDelta.maplet>

This maplet can also be run via MapleNet at the URL:

<http://maplenet.math.sc.edu/maplenet/141L-F04/CalcUSC/EpsilonDelta.html>

- If you want more precise solutions to an equation (or inequality), use Maple's `solve` command. The general syntax is

```
solve( eqn, var );
```

where *eqn* is the equation or inequality and *var* is the variable to be solved for. For example, the command `solve(abs(sqrt(x)-2)<0.05, x);` can be used to show that the inequality $|\sqrt{x} - 2| < 0.05$ is satisfied for all x in the interval (3.8025, 4.2025).

- Note that the `fsolve` command is needed if the equation is sufficiently complicated. Because the `fsolve` command does not accept inequalities, inequalities must be converted to equalities. For example, to find the endpoints of the interval where $|\sqrt{x} + x^2 - 2| < 0.2$, use `fsolve(sqrt(x)+x^2 - 2 = 0.2, x);` and `fsolve(sqrt(x)+x^2 - 2 = -0.2, x);`.

Preparation

Review the precise definition of the limit (pages 138–142 in Anton).

Activities

- Start a Maple session.
- Launch the *EpsilonDelta* maplet.
- Follow the TA's discussion of Exercises #3, 5, and 9 from §2.4 (page 145) of Anton.
- For each of the following limits and values of ϵ , find a number δ such that $|f(x) - L| < \epsilon$ if $0 < |x - a| < \delta$.
 - $\lim_{x \rightarrow 9} \sqrt{x} = 3$, $\epsilon = 0.15, 0.1, \text{ and } 0.05$. [#16, p. 145]
 - $\lim_{x \rightarrow 2} (x^2 + 3x - 1) = 9$, $\epsilon = 1, 0.8, \text{ and } 0.6$. [#32, p. 145]
 - $\lim_{x \rightarrow 3} (4x - 5) = 7$, $\epsilon = 0.8, 0.4, \text{ and } 0.2$. [#20, p. 145]

For 3. identify a rule that could be used to select δ for any value of ϵ .

Assignment

Maple Quiz 1 will be given this week. Your TA will provide the quiz questions and the due date.