MATH 241 SECTION H02
SPRING 2020

Time: Monday, Wednesday, and Friday 1:10 p.m. to 2:00 p.m.
Place: 112 LeConte

Instructor: George F. McNulty
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Office Hours: 11:45 a.m. to 1:00 p.m. Monday, Wednesday, and Friday

Free Tutoring Student Success Center: Thomas Cooper Library Mezzanine
`http://www.sc.edu/tutoring`

Text: Thomas' Calculus, 13^{th} Edition
Author: George B. Thomas et. al.
We will cover most material in chapters 12–16.

Last day to drop a course without extenuating circumstances: Wednesday 28 March 2020.

Midterm Exams: Friday 21 February
Friday 20 March
Friday 17 April

Final Exam: Wednesday 29 April 2020 at 12:30 pm

Please inform me as soon as possible if any of these times is a problem.
Alternatives can be arranged.

The main outcome of your work in our course should be a mastery of differentiation and integration of vector-valued functions of several variables and the acquisition of an understanding of the algebra and geometry of low dimensional vector spaces. By the end of the course, each diligent student should be in a position to understand, for example, the mathematical content of Maxwell's Equations describing electromagnetic radiation. The attached sample final gives a detailed view of the skills and understanding you should have by the end of the course.

While I plan to give lectures, some of our time in class will be spent in discussion and working in small groups. For this reason, active personal participation is a key to the course. Your attendance and efforts will be needed during every meeting of the class.

Homework is at the heart of our course. Generally, an assignment will be due at the beginning of every class. Homework will be collaborative. The class will be divided into small teams for the purposes of homework.

Every one of you is welcome to come to my office at anytime. I will generally be in every day from 10 am until 5 pm. While I have other responsibilities, your success is my first priority. Most of the time I will be able to set aside whatever I am doing, so don't hesitate to visit my office.

I hope you will find our course enjoyable, informative, and useful.

How Course Grades Will be Determined

The objectives of this course can be broken down into 12 sorts of problems. Samples of these 12 sorts are attached below. The course Final will resemble this collection of sample problems. In turn these 12 sorts fall into two categories: core problems and those which lie outside the core. Your grade for the course will be determined by how well you display mastery of these problems. For each sort of problem I identify three levels of performance: master level, journeyman level, and apprentice level. The examinations given during the course provide you with opportunities to display your level of performance. These examinations will all be cumulative. The First Midterm will probably have 4 problems, the Second 8 problems (with 4 being variants of the ones occurring on the First Midterm), the Third Midterm will probably have all 12 problems, and there will a few quizzzes, each offering an opportunity to each student to do a problem of his or her own choosing. I record how well you do on each problem (an M for master level, a J for journeyman level, an A for apprentice level) on each exam and on each quiz. After the Final, I make a record of the highest level of performance you have made on each sort of problem and use this record to determine your course grade. If you have at some point during the semester displayed a mastery of each of the 12 sorts of problems, then your grade will be an A. If you have at some point during the semester displayed a mastery of each of the core problems, then you will get at least a C. The grade B can be earned by displaying mastery of all the core problems and mastery of about half of the rest of the problems. The grade D will be assigned to anyone who can master several problems but has not yet displayed a mastery of all the core problems. In borderline cases, the higher grade will be assigned to those students who turn in their homework regularly.

For time to time we will have in-classes quizzes. It is also possible to arrange out-of-class quizzes. These quizzes will have just one problem of your choice.

This particular way of arriving at the course grade is unusual. It has some advantages. Each of you will get several chances to display mastery of almost all the problems. Once you have displayed mastery of a problem there is no need to do problems like it on later exams. So it can certainly happen that if you do well on the midterms you might only have to do one or two problems on the Final. (It is not unusual that a few students do not even have to take the final.) On the other hand, because earlier weak performances are not averaged in, students who come into the Final on shaky ground can still manage to get a respectable grade for the course.

This method of grading also stresses working out the problems in a completely correct way, since accumulating a lot of journeyman level performances only results in a journeyman level performance. So it pays to do one problem carefully and correctly as opposed to trying get four problems partially correctly. Finally, this method of grading allows you to see easily which parts of the course you are doing well with, and which parts deserve more attention.

The primary disadvantage of this grading scheme is that it is complicated. At any time, if you are uncertain about how you are doing in the class I would be more than glad to clarify the matter.

Sample Final Examination
MATH 241 Section H02
Spring Semester 2020

Instructor: Prof. George McNulty

Problem 0.
Let $\mathbf{u} = \langle 1,2,3\rangle, \mathbf{v} = \langle 3,2,1\rangle,$ and $\mathbf{w} = \langle -1,0,1\rangle$. Perform the calculation in each part below.

a. $2\mathbf{u} - 3\mathbf{v}$.
b. $\mathbf{u}\cdot\mathbf{v}$
c. $|\mathbf{w}|$
d. $\mathbf{u}\times\mathbf{v}$

Problem 1.
In each part below let $\mathbf{u}$ and $\mathbf{v}$ be any vectors in three dimensional space.

a. Explain why $|\mathbf{u}+\mathbf{v}|^2 + |\mathbf{u}-\mathbf{v}|^2 = 2|\mathbf{u}|^2 + 2|\mathbf{v}|^2$.
b. Explain why $(2\mathbf{u}+3\mathbf{v})\cdot(\mathbf{u}\times\mathbf{v}) = 0$

Problem 2 (Core).

a. Suppose C is the curve on the plane described by $\mathbf{r}(t) = \sin t\mathbf{i} + 2\cos t\mathbf{j}$. Then the point $\langle\sqrt{2})/2, \sqrt{2}\rangle$ lies on the curve C. (What is an appropriate choice for t?). Find an equation that describes the line tangent to C at $\langle\sqrt{2})/2, \sqrt{2}\rangle$.
b. Find an equation that describes the plane which is determined by the points $\langle 0,2,1\rangle, \langle 0,2,-1\rangle,$ and $\langle 2,0,0\rangle$.

Problem 3 (Core).
Let $\mathbf{F}(t) = \sin t\mathbf{i} + t^2\mathbf{j} + 2\mathbf{k}, \mathbf{G}(t) = t^2\mathbf{i} + \cos t\mathbf{j} + e^t\mathbf{k},$ and $h(t) = 4t$. Calculate the derivative each function below.

a. $h(t)\mathbf{F}(t)$.
b. $\mathbf{F}(h(t))\cdot\mathbf{G}(t)$.
c. $\mathbf{F}(t)\times\mathbf{G}(t)$.

Problem 4 (Core).
In each part below find an equation for the plane tangent to the surface it describes at the point given.

a. $x^2 + y^2 + z^2 = 3$ at $\langle 1,1,1\rangle$.
b. $z = \ln(x^2+y^2)$ at $\langle 1,0,0\rangle$.

Problem 5.
Use the Chain Rule to complete each part below.

a. Calculate $\dfrac{dw}{dt}$ at $t=0$ where $w = x^2+y^2, x = \cos t + \sin t,$ and $y = \cos t - \sin t$.
b. Express $\dfrac{\partial z}{\partial r}$ and $\dfrac{\partial z}{\partial \theta}$ as functions of r and θ where $z = 4e^x \ln y, x = \ln(r\cos\theta),$ and $y = r\sin\theta$.

PROBLEM 6.

Complete each part below.

a. Calculate the gradient of $f(x,y,z) = x^2 + y^2 - 2z^2 + z\ln x$ at $\langle 1,1,1\rangle$.

b. Calculate the derivative of $f(x,y,z) = 3e^x \cos yz$ at $\langle 0,0,0\rangle$ in the direction of $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

PROBLEM 7 (CORE).

In each part below find all the local critical points. For each extreme point, determine whether it is a local minimum, a local maximum, or a saddle point.

a. $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$

b. $g(x,y) = \dfrac{1}{x} + xy + \dfrac{1}{y}$

PROBLEM 8 (CORE).

Do each part below.

a. Evaluate $\iint\limits_D x + 2y\,dA$ where D is the region bounded by the parabolas described by $y = 2x^2$ and $y = 1 + x^2$.

b. Find the volume of the solid bounded by the plane described by $z = 0$ and the paraboloid described by $z = 1 - x^2 - y^2$.

PROBLEM 9 (CORE).

Do each part below.

a. Evaluate $\int\limits_C 2x + 9z\,ds$ where C is the curve parameterized by $x = t, y = t^2$, and $z = t^3$ for $0 \le t \le 1$.

b. Evaluate $\int_C e^x \sin y\,dx + e^x \cos y\,dy$, where C is any curve connecting $(0,0)$ to $(1, \pi/2)$.

PROBLEM 10.

Evaluate the integral in each part below.

a. $\oint_C 3y - e^{\sin x}\,dx + (7x + \sqrt{y^4+1})\,dy$ where C is the circle described by $x^2 + y^2 = 9$.

b. Let $\mathbf{F} = (x^2+y^2)\mathbf{i} + 2xy\mathbf{j}$ and let C be the boundary of the unit square with vertices at $(0,0), (1,0), (1,1)$, and $(0,1)$. Evaluate $\oint_C \mathbf{F}\cdot\mathbf{T}\,ds$.

PROBLEM 11.

Find the surface area of that part of the cylinder described by $y^2 + z^2 = 9$ that is directly over the rectangle in the XY-plane with vertices $(0,0), (2,0), (2,3)$, and $(0,3)$.