An example of using Representation Theory to find a resolution

Let R be a polynomial ring. A resolution is a sequence of matrices d_1, d_2, $\dots$, with entries in R, such that the kernel of d_i is equal to the image of d_{i+1} for all i. The problems "Find a resolution" or "Show that this is a resolution" appear to require one to solve large systems of equations over R.

In this talk we will use Representation Theory to create some resolutions. The Representation Theory works like magic! (That is, we will never come even close to solving any equations.) The Representation Theory will show both that our candidates are complexes (i.e., $\operatorname{im} d_{i+1} \subseteq \ker d_i$) and that the complexes are exact (i.e., $\ker d_i \subseteq \operatorname{im} d_{i+1}$).

The method is based on the Acyclicity Lemma from Commutative Algebra and the Littlewood-Richardson rule from Representation Theory. We will not prove these results in this seminar; but, we will show how to use them. Indeed, the main proofs that we will show involve the Combinatorics of moving boxes around inside Young Diagrams.

The modules that we resolve are maximal Cohen-Macaulay modules over a determinantal ring. The resolutions that we produce include the family of Eagon-Northcott resolutions.