Mathematics 700 Homework
Due Wednesday, September 3

(1) Let V be a vector space over $\mathbf{F}$ and $S \subset V$ a non-empty subset of V. Show that S is a subspace of V if and only if $S = \operatorname{Span}(S)$.

(2) Which of the following is a subspace of $\mathbf{R}^3$. Give a brief justification of your answer.
- (a) $\{(x, y, z) : 2x + 3y + 4z = 0\}$
- (b) $\{(x, y, z) : 2x + 3y + 4z = 1\}$
- (c) The line through $(1, 2, 3)$ and parallel to $(1, 0, 4)$.
- (d) $\{(x, y, z) : x, y, z \geq 0\}$

(3) Let V be the vector space of all continuous function $f : \mathbf{R} \to \mathbf{R}$. Which of the following subsets of V are subspaces? Give a brief justification of you answer.
- (a) $\{f : f(0) = 2f(5)\}$
- (b) $\{f : f(x^2) = f(x)^2\}$
- (c) $\{f : \int_0^1 f(x)\,dx = 0\}$
- (d) $\{f : \int_0^1 f(x)\,dx = 1\}$

(4) Let V be a vector space and $U, W \subset V$ subspaces of V. Show that $U \cup W$ is a subspace of V if and only if $U \subseteq W$ or $W \subseteq U$.

(5) Let V be a vector space and $v_1, \ldots, v_m \in V$. Show that $v_1, \ldots, v_m$ are linearly dependent if and only if one of $v_1, \ldots, v_m$ can be written as a linear combination of the others. (This is a standard result and will be used often later in the term.)

(6) Let V be a vector space and $v_1, \ldots, v_m \in V$. Show that $v_1, \ldots, v_m$ are linearly independent if and only if any $v \in \operatorname{Span}\{v_1, \ldots, v_m\}$ has a unique expression as a linear combination of $v_1, \ldots, v_m$. (Another standard result will be used repeatedly.)

(7) Let $\mathcal{P}_3$ be the vector space of polynomials of degree ≤ 3. Which of the following sets are linearly independent. Justify your answer.
- (a) 1, x, x^3.
- (b) x^3, $(x+1)^3$, $(x+2)^3$.
- (c) x^2, $(x+1)^2$, $(x+2)^2$, $(x+3)^2$.

The First Quiz.

The first quiz will be on Wednesday September 3 and will cover Chapter 3 of *Schaum's Outline* (systems linear of equations). Know the following:
- (a) The three elementary operations E_1, E_2, and E_3 on paper 63.
- (b) Theorem 3.4 on page 63 on the equivalence of systems.
- (c) Section 3.6 on Gaussian elimination pages 69–73