Mathematics 700 Homework Due Wednesday August 25

The first quiz will be on Wednesday September 1 and will cover Chapter 1 of *Schaum's Outline* (linear systems of equations). In know the following

1. Theorem 1.4 page 7.
2. The reduction algorithm on page 11.
3. Theorem 1.7 page 12 (this is only true when the field of scalars $\mathbf{F}$ is infinite.)
4. Theorem 1.8 page 16 (this is important)
5. Theorem 1.10 page 19 (also important)
6. Theorem 1.11 page 20 (another important one).

Problems to be collected

1. Prove the following: Let $\mathbf{F}$ be a field. Then for all $a, b \in \mathbf{F}$
   (a) $a \cdot 0 = 0$
   (b) $ab = 0$ if and only if $a = 0$ or $b = 0$.
   (c) $x^2 = a^2$ implies $x = a$ or $x = -a$.
   (d) If $ad - bc \neq 0$ then
   $$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned} \quad \text{implies} \quad \begin{aligned} x &= \frac{ed - fb}{ad - bc}, \\ y &= \frac{af - ce}{ad - bc}. \end{aligned}$$
   (e) If $b^2 - 4ac$ has a square root in $\mathbf{F}$ (that is here is an element $\alpha \in \mathbf{F}$ so $\alpha^2 = b^2 - 4ac$) then $a \neq 0$ implies
   $$ax^2 + bx + c = 0 \quad \text{implies} \quad x = \frac{-b \pm \alpha}{2a}.$$
   (That is the usual quadratic formula $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$ holds in $\mathbf{F}$ provided that the square root $\sqrt{b^2 - 4ac}$ exists as an element of $\mathbf{F}$.) Hint: Complete the square just as in the usual derivation of the quadratic formula.
2. Let V be a vector space and $\{W_\alpha : \alpha \in A\}$ collection of subspaces of V. Then show that the intersection
   $$W = \bigcap_{\alpha \in A} W_\alpha$$
   is also a subspace of V. (We are not assuming that the collection $\{W_\alpha : \alpha \in A\}$ is finite.)