

Mathematics 522 Test #2

Name: _____

Show your work! Answers that do not have a justification will receive no credit.

1. (40 Points) Compute the following

(a) The derivative of $e^z \sin(z^2 + z)$

(b) $\log(-7 + 7i)$

(c) $\cos(2 + 3i)$

(d) $(2i)^i$

(e) All solutions to $\cos(z) = 3$

2. (15 Points) If f is analytic on the open set D show that $\operatorname{Re} f$ is harmonic on D .

3. (15 Points) Find the harmonic conjugates of $u = e^{2x} \sin(2y) - 4xy + y$

4. (15 Points) Let f be a complex valued function on a domain D so that $f'(z) = 0$ for all $z \in D$. Then show that $f(z)$ is constant in D .

5. (15 Points) Recall that the derivative of $\tan(z)$ is $\frac{d}{dz} \tan(z) = \sec^2(z) = 1 + \tan^2(z)$. Use this to show that the derivative of $w = \arctan(z)$ is $w' = \frac{1}{1 + z^2}$.