

Mathematics 522 Test #1

Name: _____

Show your work! Answers that do not have a justification will receive no credit.

1. (25 Points) Compute the following:

(a) $(3 + 4i)(5 - 2i)$

(b) $\frac{3 + 4i}{5 - 2i}$

(c) $(1 - \sqrt{3}i)^{14}$

(d) $\left| \frac{(3 + 4i)^4}{(3 - 4i)^5} \right|$

(e) $e^{\frac{5\pi}{6}i}$

(f) $\arg(-\sqrt{3} + i)$

(g) $\operatorname{Re}[(2 + 3i)(x + yi)]$

2. (10 Points) Find all cube roots of -27 .

3. (15 Points) Show that for real numbers x and y that $|e^{x+iy}| = e^x$.

4. (10 Points) What is the image of the disk $|z - i| < 2$ under the map $f(z) = (3 + 4i)z + 2$?
Draw pictures.
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5. (10 Points) Draw pictures of the following sets of complex numbers.
(a) $|z - 2 + 3i| < 4$

(b) $1 < |z| < 4$

6. (20 Points) Let D be the domain defined by $|z| < 3$ and $0 < \text{Arg}(z) < \pi/2$ and let h be the function $h(z) = 2z^3$.
- (a) Draw a picture of D .

- (b) Find the image $h[D]$ and draw a picture of it.

7. (10 Points) Let $f: \mathbb{C} \rightarrow \mathbb{C}$ be a complex valued function.

(a) State the definition of what it means for f to be differentiable at z_0 .

(b) Using the definition of being differentiable find the derivative of $f(z) = z^3$.