Test 1

Name: _____

Show your work! Answers that do not have a justification will receive no credit.

1. (10 points) Solve $z^2 + (2+3i)z - 1 + 3i = 0$ Answer: _____

2. (10 points) Prove $\mathrm{Im}((4+3i)z) = 3\,\mathrm{Re}(z) + 4\,\mathrm{Im}(z)$.

3. (15 points) (a) Find $(1 - \sqrt{3}\,i)^9$. Answer: _____

(b) Find all cube roots of $8i$. Answer: _____

4. (20 points) (a) Show that for any real number y that $|e^{iy}| = 1$.

(b) Show that for any complex number z that $|e^z| = e^{\operatorname{Re} z}$.

5. (15 points) Use the relation $e^{(\alpha+\beta)i} = e^{\alpha i}e^{\beta i}$ to derive the addition formal for the sine function.

6. What is the image of the circle $|z| = 1$ under the map $f(z) = 2iz + 3$?

7. (20 points) Let D be the domain defined by the conditions $0 < |z| < 2$ and $|\operatorname{Arg}(z)| < \frac{\pi}{4}$.

(a) Graph the domain D.

(b) Let $f(z) = z^3$. Then find and graph the image of D under f.