Mathematics 551 Test #1 Name: _____

Show your work! Answers that do not have a justification will receive no credit.

(1) (5 Points) State the Frenet formulas for a C^2 regular unit speed curve in $\mathbb{R}^2$ carefully defining all the quantities involved.

(2) (5 Points) Let $\alpha\colon [a,b]$ be a C^2 regular curve in $\mathbb{R}^2$, but we do not assume that α is unit speed. Then, using the Frenet formulas define formulas for the velocity vector $\dfrac{d\alpha}{dt}$ and the acceleration vector $\dfrac{d^2\alpha}{dt^2}$ in terms of the speed v, curvature κ, unit tangent $\mathbf{t}$ and unit normal $\mathbf{n}$ of α.

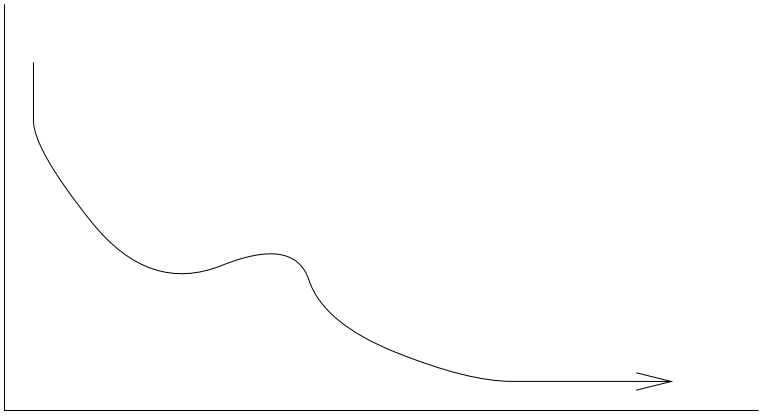
(3) (10 points)

(a) Parameterize the ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$.

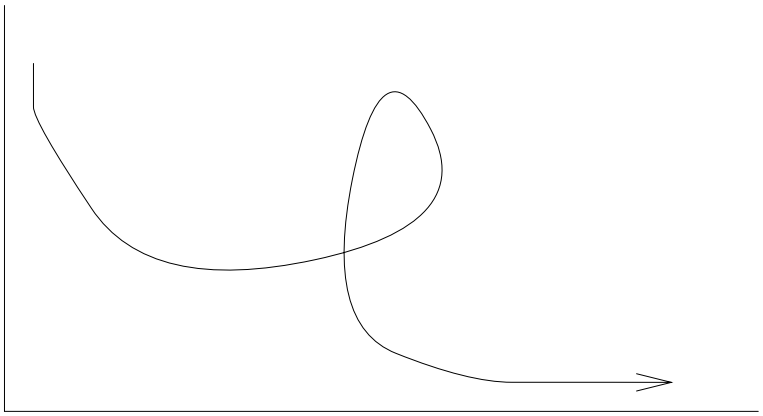
(b) Set up the integral for the length of this ellipse (do not evaluate this integral.)

(4) (10 points) Find the curvature of the curve $\alpha(t) = (e^t \cos(t), e^t \sin(t))$

(5) (10 points) For the following cureves find $\int_0^L \kappa(s)\,ds$.



$$\int_0^L \kappa(s)\,ds = \underline{\qquad\qquad\qquad\qquad}$$



$$\int_0^L \kappa(s)\,ds = \underline{\qquad\qquad\qquad\qquad}$$