

Final

Name: _____

Show your work! Answers that do not have a justification will receive no credit.

1. (15 points)

(a) Negate the following two statements:

i. All good students do fine.

ii. Either the parallel postulate holds, or some triangle has angle sum 30° .

(b) Give the converse of: P implies Q

(c) Give an example of valid implication whose converse is not valid.

2. (20 points) True or false and give a short reason for your answer.

(a) Using just the incidence axioms it is possible to prove that all lines have at least four points.

(b) If $A * B * C$ and $C * D * E$, then $A * D * E$

(c) Two distinct lines ℓ and m can intersect in at most one point.

(d) Two distinct rays \overrightarrow{r} and \overrightarrow{q} can intersect in at most one point.

3. (15 points)

(a) Give a model of the incidence axioms where the parallel postulate is true.

(b) Give a model of the incidence axioms the parallel postulate is false.

(c) Give a brief explanation of why it is impossible to prove the parallel postulate from the incidence axioms.

4. (30 points) Prove PASCH'S THEOREM: If A , B and C are distinct noncollinear points and ℓ is a line intersecting \overline{AB} at a point between A and B , then ℓ also intersects either \overline{AC} or \overline{BC} .

5. (30 points) Prove that the base angles of an isosocles triangle are congruent.

6. (30 points) Let $\triangle ABC$ have $\overline{AB} \cong \overline{AC}$ and let M be the midpoint of \overline{BC} . Then prove $\triangle ABM$ is a right triangle.

7. (30 points) Prove ANGLE SUBTRACTION: Given \overrightarrow{BG} between \overrightarrow{BA} and \overrightarrow{BC} , \overrightarrow{EH} between \overrightarrow{ED} and \overrightarrow{EF} , $\sphericalangle CBG \cong \sphericalangle FEH$, and $\sphericalangle ABC \cong \sphericalangle DEF$. Then $\sphericalangle GBA \cong \sphericalangle HED$.

8. (30 points) Prove that if a rectangle exists, then there is a triangle that has angle sum 180° .