Mathematics 527 Test #3 Name: _____

Show your work to get credit. An answer with no work will not get credit.

(1) (5 points) State the difference formula relating $f[x_0, x_1, \ldots, x_n]$ and $f[x_0, x_1, \ldots, x_{n-1}]$ and $f[x_1, x_2, \ldots, x_n]$.

(2) (5 points) How are n order divided differences $f[x_0, x_1, \ldots, x_n]$ related to the n-th derivative $f^{(n)}$.

(3) (5 points) State the intermediate value theorem.

(4) (20 points) Let $\phi(h)$ be a functions so that

$$\phi(h) = L + a_6 h^6 + a_8 h^8 + a_{10} h^{10} + \cdots$$

Then find a function ψ so that

$$\psi(h) = L + b_8 h^8 + b_{10} h^{10} + \cdots$$

for some constants b_8 and b_{10} and give the relationship between a_8, a_{10} and b_8 and b_{10}.

(5) (15 points) What is the error term in the approximation

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

(6) (10 points) If g is continuous on $[a, b]$ and $x_1, x_2, x_3 \in [a, b]$ the explain why there a $\xi \in [a, b]$ such that

$$2f(x_1) + 3f(x_2) + 4f(x_3) = 9f(\xi).$$

(7) (15 points) An upper sum with n equally spaced points is used to approximate $\int_0^2 \sqrt{1 + x^3}\, dx$. How large do we need to take n insure the error is less than .01?

(8) (15 points) How large must n be chosen in the composite trapezoid rule to insure that the error in computing $\int_0^3 \frac{dx}{1+x}$ is less than .001?

(9) (10 points) Express $\int_0^\alpha \frac{\sin x}{x}\, dx$ as a series in α