Mathematics 527 Test #1 Name:____________________________

Show your work to get credit. An answer with no work will not get credit.

(1) (5 points) State the n-th order Taylor theorem about x and with remainder for $f(x+h)$.

(2) (5 points) State the mean value theorem.

(3) (10 points) How many terms of the Taylor series for $e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}$ are needed to compute $\sqrt{e} = e^{.5}$ to 5 decimal places? Explain your answer.

(4) (10 points) Let

$$\alpha = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}.$$

How many terms of this series is needed to compute α to 10 decimal places? Explain your answer.

(5) (15 points) Define the following:

(a) $x_k \to r$ ***linearly***.

(b) $x_k \to r$ ***quadratically***.

(c) r is a ***fixed point*** of $g(x)$.

(6) (10 points) The bisection method is used to solve for a root of $f(x) = 0$ in the initial interval $[5, 10]$. How many steps are needed to find the root accurate to 8 decimal places? Explain your answer.

(7) (15 points) Let $g\colon [a, b] \to [a, b]$ be differentiable with $|g'(x)| \leq .1$ and we wish to solve the equation $g(x) = x$. Let r be the root, let $x_0 = (b - a)/2$ be the midpoint of $[a, b]$, and define

$$x_1 = g(x_0), \quad x_2 = g(x_1), \quad x_3 = g(x_2), \ldots$$

(a) Show that $|r - x_k| \leq .1|x_{k-1} - r|$. HINT: Mean value theorem.

(b) Explain why $|r - x_k| \leq (.1)^k(b - a)/2$.

(8) (15 points) In the following figure we start Newton's method at some initial point x_0 and form the points $x_0, x_1, x_2 \ldots$ in the usual manner. For the following choices of x_0 if Newton's method will converge, and if so to what point it will converge.

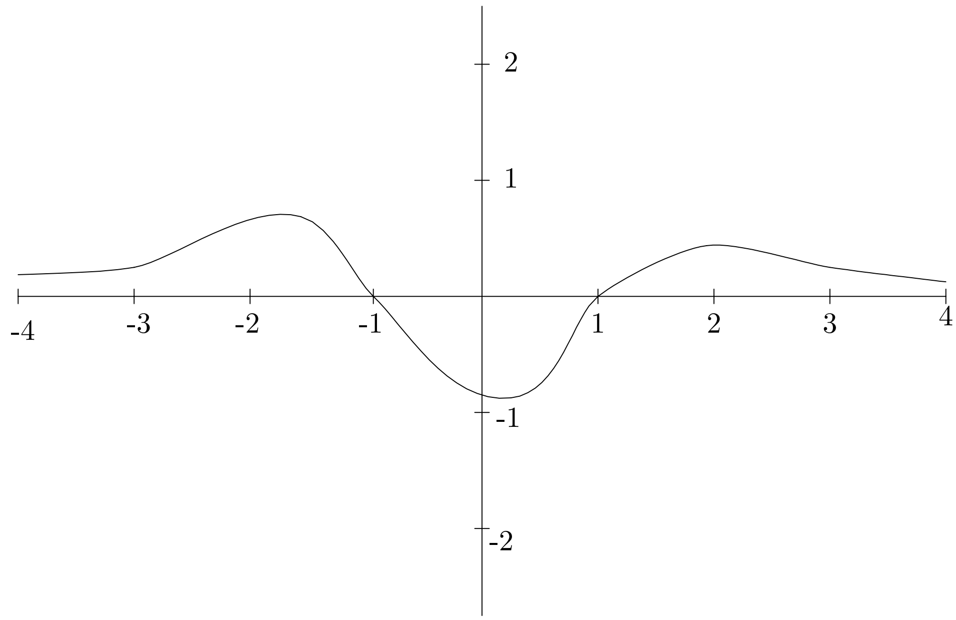

FIGURE 1

(a) $x_0 = .75$

(b) $x_0 = -.5$

(c) $x_0 = 3.$

(9) (10 points) If we have a sequence x_k from an application of Newton's method to find the root of r of $f(x) =$, so that the errors $e_k = r - x_k$ satisfy $|e_{k+1}| \leq (.1)e_k^2$ and the initial error $e_0 \leq 1$, then how many steps are needed to commute r accurate to 50 decimal places?

Number of steps = ____________________

(10) (5 points) State the quotient remainder theorem for polynomials.