Math/Stat 511 Test #2 Name: Answer Key

Show your work! Answers that do not have a justification will receive no credit.

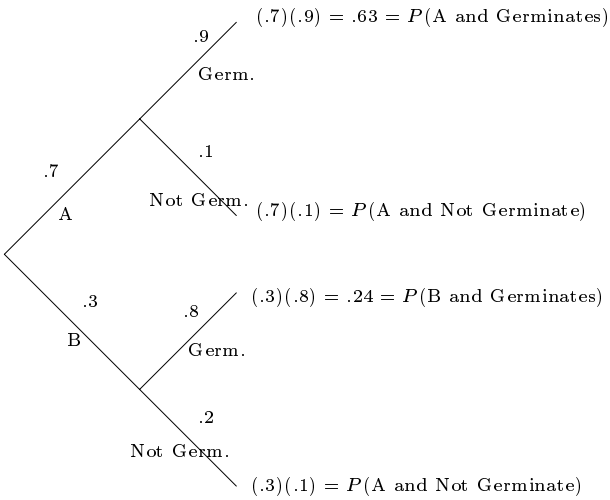
Grades on the Second Exam.

Here is the information on the second test. 18 people took the exam. The high score was a 100 and three were 99's. The two low scores were 29 and 34 The mean was $\mu = 78.05$ with a standard deviation of $\sigma = 22.17$. The median was 84.00 The break down in the grades is in the table.

Grade	Range	Number	Percent
A	90–100	7	38.89%
B	80–89	3	16.67%
C	70–79	2	11.11%
D	60–69	3	16.67%
F	0–59	3	16.67%

1. (10 Points) Corn seeds from supplier A have a 90% germination rate and those from supplier B have a 80% germination rate. A package of corn seeds has 70% of its seeds from supplier A and 30% from supplier B. If a seed from this package is planted and germinates, then what is the probability that it came from supplier B?

   Solution: This can be done with Bayes' Law, but it is easier to use a tree diagram.



   Therefore

$$\begin{aligned} P(B|\text{Germinates}) &= \frac{P(\text{A and Germinates})}{P(\text{Germinates})} \\ &= \frac{(.3)(.8)}{(.7)(.9)+(.3)(.8)} \\ &= \frac{.24}{.87} \approx \underline{.275862} \end{aligned}$$

   Remark: The answers $.87 = P(\text{Germinates})$ and $.24 = P(B|\text{Germinates})$ both received 3 out of the 10 points.

2. (10 Points) Let X be a random variable so that the the p.d.f. of X is given by

$$f(x) = cx, \qquad x = 2, 3, 4.$$

   for some constant c.

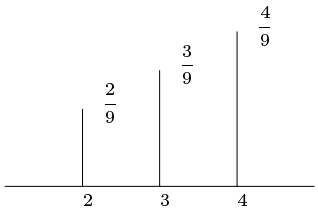
   (a) Find c $\qquad c =$ _____

   Solution: As the sum of a *p.d.f* is 1 we have

$$1 = f(2) + f(3) + f(4) = c \cdot 2 + c \cdot 3 + c \cdot 4 = 9c$$

   which implies $c = \underline{1/9}$.

   (b) Depict the p.d.f. as a bar graph.



3. (5 Points) In a state lottery a two digit number is chosen at random. If player bets \$1 on a particular number, then he or she wins \$75, otherwise he or she loses \$1. What is the expected payoff for this game?

   Solution: Let X be the payoff. Then X takes on the two values $X = -1\$1$ and $X = \$75$. The p.d.f. is $f(-1) = .99$ and $f(75) = .01$. Therefore the expected payoff is

$$E(X) = (-1)f(-1) + 75f(75) = (-\$1)(.99) + \$75(.01) = \underline{-\$.24}.$$

   Remark: As there was a little ambiguity in the statement the answer where if was assume that X takes on the values $X = -1$ and $X = 74$ with $f(-1) = .99$ and $f(74) = .01$

$$E(X) = (-1)f(-1) + 74f(74) = (-\$1)(.99) + \$74(.01) = \underline{-\$.25}.$$

   also got full credit.

4. (10 Points) Let X be a discrete random variable with p.d.f.

$$f(x) = \frac{5-x}{10}, \qquad x = 1, 2, 3, 4.$$

   Find the mean and variance of X.

   Solution: It is a little easier to see what is going on if we make a table for the p.d.f.

x	1	2	3	4
$f(x)$	.4	.3	.2	.1

   Then

$$\text{mean} = \mu = E(X) = 1f(1) + 2f(2) + 3f(3) = 4f(4) = 1(.4) + 2(.3) + 3(.2) + 4(.1) = \underline{2.0}$$

$$\begin{aligned}\text{variance} = \sigma^2 &= E[(X-\mu)^2] = E[(X-2)^2] \\ &= (1-2)^2 f(1) + (2-2)^2 f(2) + (3-2)^2 f(3) + (4-3)^2 f(4) \\ &= 1(.4) + 0 + 1(.2) + 4(.1) \\ &= \underline{1}\end{aligned}$$

5. (15 Points) Let X be the value of a number chosen at random from the set $\{10, 11, \ldots, 20\}$ with all numbers equally likely.

   (a) What is the p.d.f. of X?

   Solution: This is the discrete uniform distribution on $\{10, 11, \ldots, 20\}$ and there are 11 points in this set so the p.d.f. is

$$f(x) = \frac{1}{11} \quad \text{for} \quad x = 10, 11, \ldots, 20.$$

   Remark: Forgetting to put $x = 10, 11, \ldots, 20$ lose one point.

   (b) What is $P(14 \le X \le 18)$?

   Solution: This if

$$P(14 \le X \le 18) = f(14) + f(15) + f(16) + f(17) + f(18) = frac511$$

   (c) What is the expected value of X?

   Solution: From the formula for the mean of a discrete uniform

$$E(X) = \frac{a+b}{2} = \frac{10+20}{2} = 15.$$

(d) What is the variance of X? $V(X) =$ _____

From the formula for the variance of a discrete uniform random variable. If $m = b - a + 1$ then

$$V(X) = \frac{m^2 - 1}{12} = \frac{11^2 - 1}{12} = \frac{121 - 1}{12} = 10.$$

6. (10 Points) A bag contains 20 pieces of candy of which 5 are cherry and 15 are grape flavored. If 6 pieces of candy are chosen at random from the bag (without replacement), then what is the probability that exactly 3 are cherry?

Solution: This is a hypergeometric distribution and the probability is

$$\frac{\binom{5}{3}}{\binom{15}{3}}\binom{20}{6} \approx \underline{.117438}$$

Remark: Not changing to a decimal lost 2 points. Thinking it was a binomial or exponential distribution lost all ten points.

7. (20 Points) In a certain state 15% of people do not have auto insurance. A random sample of 10 people is made and the number, X, of people who do not have auto insurance is recorded.

(a) What is the distribution of X.

Solution: This is a binomial distortion $b(10, .15)$. (That is $n = 10$ and $p = .15$)

(b) What is the expected number of people in the sample that do not have auto insurance.

Solution: This is the mean of X which is $E(X) = np = 10(.15) = \underline{1.5}$.

(c) Compute the following probabilities.

Solution: Use the table for the binomial distribution.

(i) $P(X \leq 3) = \underline{.9500}$

(ii) $P(X \geq 4) = 1 - P(X \leq 3) = 1 - .9500 = \underline{.0500}$

(iii) $P(X = 2) = P(X \leq 2) - P(X \leq 1) = .8202 - .5453 = \underline{.2759}$

8. (15 Points) A student takes a multiple choice test where the probability of his getting a right answer by guessing is $p = .2$. Assume that he guesses on all the questions and that the guesses are independent

(a) What is the probability that his fist correct answer is on question 6?

Solution: This is an exponential distribution with $p = .2$. Thus

$$P(\text{first correct answer on question 6}) = (.8)^5(.2) \approx \underline{.065536}$$

(b) What is the probability that his third correct answer is on question 12.

Solution: This is a negative binomial distribution with $p = .2$ and $r = 3$. Therefore

$$P(\text{ third correct answer is on question 12}) = \binom{11}{2}(.8)^9(.2)^3 \approx \underline{.0590558}$$

(c) What is the probability he gets the first 5 questions wrong?

Solution: This is just $(.8)^5 = \underline{.32726}$

(d) Let X be the number of the question on which he gets his fourth correct answer. Then what are the mean and variance of X.

Solution: Here X has the negative binomial distribution with $r = 4$ and $p = .2$. From the formula for the mean and variance

$$E(X) = \frac{r}{p} = \frac{4}{.2} = \underline{20}$$

and

$$V(X) = \frac{rq}{p^2} = \frac{4(.8)}{(.2)^2} = \underline{80}$$

9. (5 Points) Let X be a random variable with expected value $E(X) = 2$ and variance $\sigma^2 = V(X) = 3$. Then compute $E[X(4 - X)]$

Solution: We know that $\mu = E(X) = 2$. Therefore $3 = V(X) = E(X^2) - \mu^2$ implies $E(X^2) = V(X) + \mu^2 = \sigma^2 + \mu^2 = 3 + 2^2 = 7$. Whence

$$E[X(4 - X)] = E(4X - X^2) = 4E(X) - E(X^2) = 4(2) - 7 = \underline{1}.$$