Test 2

Name: ______________________

Show your work! Answers that do not have a justification will receive no credit.

1.(40 points) Solve the following equations. If no initial values are given find the general solution. If initial values are given then solve the initial value problem.

(a) $y'' - 8y' + 25y = 0.$

(b) $y'' + 4y' - 8y = 0,$

(c) $y'' - 10y' + 25y = 0, \qquad y(0) = 3,\ y'(0) = -4$

(d) $(1+x^2)y' - 2xy = (1+x^2)^2, \qquad y(0) = 1.$

2. (10 points) Let y satisfy the equation $y' + py = qy^4$ where p and q are continuous functions. Find the value of the constant α is so that the substitution $y = u^\alpha$ makes the equation $y' + py = qy^4$ first order linear.

3. (20 points) In this problems we consider solutions of the equation $y' = y(4 - y^2)$.

(a) Find stationary solutions, that is the solutions that are constant with respect to time.

(b) Graph the solutions with the following initial conditions $y(0) = -3$, $y(0) = -1$, $y(0) = 1$, $y(0) = 3$. Put this all on one graph.

(c) If $y(0) = .79$ what is a good approximation of $y(35)$?

4. (10 points) Show that if y_1 and y_2 are solutions to $y'' + py' + qy = 0$ and c_1 and c_2 are constants then $y = c_1y_1 + c_2y_2$ is also a solution.

5. (20 points). A very tall rectangular water tank has its base in the shape square 2 meters on a side. If water is pumped into the tank at a rate of $.2\text{m}^3/\text{sec}$ and there is a hole in the bottom of the tank that is $.1\text{m}^3$. Assume that water drains out of the hole at by Torricelli's law.

(a) Let $y(t)$ be the depth of the water in the tank after t seconds. Write down a differential equation for $y(t)$.

(b) What is the depth after the water has been running for several hours.