Final

Name:__________________________

Show your work! Answers that do not have a justification will receive no credit.

1. (60 points) Solve the following equations. If initial conditions are given then solve the initial value problem. If none are given find the general solution.

(a) $y''(t) = 12t + 24t^2, \quad y(0) = 4,\ y'(0) = -3.$

(b) $\dfrac{dV}{dr} = \dfrac{1-2r}{3V^2+2}.$

(c) $xy' - 2y = 6x^5 - 2x^4, \quad y(1) = 9.$

(d) $y'' + 4y' + 4y = 0, \quad y(0) = 3,\ y'(0) = -4.$

(e) $y'' - 4y' + 13y = 0.$

(f) $y'' - 3y' + 2y = -4x + 16\cos(2x).$

2. (15 points) The functions $y_1 = x$ and $y_2 = x^{-1}$ are solutions to $x^2y'' + xy' - y = 0$. Find the general solution to $x^2y'' + xy' - yy = 3x$

3. (15 points) The function $y_1 = x^2$ is a solution to $x^2y'' - 3xy' + 4y = 0$. Find the general solution to this equation.

4. (15 points) The roots of the characteristic equation to

$$y^{(5)} - 3y^{(4)} + 2y'' + 8y' - 8y = 0$$

are $r = 1, 2, 2, -1+i, -1-i$.

(a) What is the general solution to this equation?

(b) What is a good guess at the form of a particular solution to the equation
$y^{(5)} - 3y^{(4)} + 2y'' + 8y' - 8y = e^{5x} + 3e^{2x} + \cos(x)$?

5. (15 points) Let $L[y] = y'' + py' + qy$.

(a) Show $L[c_1y_1 + c_2y_2] = c_1L[y_1] + c_2L[y_2]$.

(b) Use part (a) to show that if $L[y_1] = 0$ and $L[y_2] = e^{x^2}$ then $y = 2y_1 + 3y_2$ is a solution to $L[y] = 3e^{x^2}$.

6. (20 points) A sailor accidently brings 10 cockroachs onto his ship in a sack of fruit just as the ship is leaving for a 20 week cruise. Assume the rate of increase of the roachs is proportional to the number of roachs on the ship. If after 2 weeks there are 50 roachs then how many are there at the end of the 20 week cruise?

7. (20 points) A tank holds 300L of water when it is full. At an initial time the tank contains 100L of water that has 10Kg of salt dissolved in it. At this time water that has 2Kg per L of salt dissolved in it is pumped in at the rate of 5L/min, and at same time water is drained out at 3L/min.
(a) How long before the tank is full?

(b) How much salt is in the tank when it becomes full?

8. (20 points) A mass of 2 Kg is attached both to a vertically suspended spring with a spring constant of $k = 26$ and a dashpot that provides 12 Newtons of resistance for every meters per second of velocity (that is the dashpot constant is $c = 12$). If the mass is pulled down 2 Meters below its static equilibrium position and released form rest,

(a) Find a formula for the position of the mass after t seconds. (Use $g = 9.8\text{m}/\text{sec}^2$).

(b) What is the position of the mass after several hours after it was released?

9. (20 points) In this problem we consider solutions of the equation $y'(t) = e^t y(t)(4 - y(t))$.

(a) Find the stationary solutions, that is the solutions that are constant with respect to time t.

(b) Graph the solutions with the following initial conditions $y(0) = -1$, $y(0) = 1$, $y(0) = 5$. Put this all on one graph.

(c) If $y(0) = 2.9$ what is a good approximation of $y(78)$?