

Mathematics 174 Test #1

Name: _____

Show your work to get credit. An answer with no work will not get credit.

- (1) (5 Points) Make a truth table for $(p \vee q) \rightarrow (p \wedge q)$.

p	q

- (2) (5 Points) Use a truth table to determine if the following argument is valid.

$$\begin{aligned} &(p \wedge q) \rightarrow r \\ &p \vee q \\ \therefore &r \end{aligned}$$

- (3) (10 Points) Rewrite the following formally (that is with symbols and labeling the variables) and determine if it is valid argument.

If Sally solved the problem correctly, then Sally obtained the answer $z = 54$.

Sally obtained the answer $z = 54$.

\therefore Sally solved the problem correctly.

Formal restatement:

Valid or invalid? _____

Justification:

(4) (15 Points) Write out the negations of the following sentences:

(a) If x is an integer, then $x(x + 1)$ is even.

(b) The sum of any two irrational numbers is irrational.

(c) Every mathematics student studies hard.

(d) For any student, if the student likes football, then this has been a good year for the student.

(e) There is a student that does not study, but gets A's.

(5) (10 Points) Define the following

(a) n is an even number.

(b) n is an odd number.

(c) n is a prime number.

(d) n is a composite number.

(e) b is a factor of n .

(6) (5 Points) Is $p \leftrightarrow \sim q$ logically equivalent to $(p \wedge \sim q) \vee (\sim p \wedge q)$. Justify your answer.

answer _____

Justification:

(7) (5 Points) Change the repeating decimal $4.545454545\dots$ to a ratio of integers.

(8) (5 Points) Change $53AD_{16}$ to base 10.

(9) (5 Points) Change 789_{10} to base 2.

(10) (35 Points) For each of the following statements say if is true or false. If true give a proof. If false give a counterexample.

(a) The sum of four consecutive integers is even.

True or False? _____

Proof or counterexample:

(b) The sum of two odd numbers is divisible by 3.

True or False? _____

Proof or counterexample:

(c) For real numbers x and y , $(x + y)^3 = x^3 + y^3$.

True or False? _____

Proof or counterexample:

(d) If n is odd then $n(n + 1)$ is even.

True or False? _____

Proof or counterexample:

(e) If $n = 2k + 3$ with k an integer, then $n^2 - 1$ is divisible by 4.

True or False? _____

Proof or counterexample: