Mathematics 174 Test #2 Name:_____

Show your work to get credit. An answer with no work will not get credit.

1. (25 Points) Evaluate the following
   (a) $27 \; div \, 4$ _____

   (b) $-32 \; div \, 7$ _____

   (c) $41 \; mod \, 11$ _____

   (d) $-43 \; mod \, 13$ _____

   (e) $\sum_{k=2}^{5}(2k^2+3)$ _____

   (f) $\prod_{m=1}^{5}\frac{2m-1}{2m+1}$ _____

   (g) $\lceil 5.48 \rceil$ _____

   (h) $\lfloor -17/3 \rfloor$ _____

2. (5 Points) Today is a Friday. What day of the week will it be 120 days from now?

   _____

3. (5 Points) Show that if $a|b$ and $a|c$, then $a|(3b-2c)$.

4. (5 Points) Show that the square of any integer is either of the form $4k$ or $4k+1$.

5. (5 Points) If $n = 6k+1$ show that 12 divides $n^2 - 1$.

6. (5 Points) If $n \, mod 3 = 2$ then show $\left\lceil \frac{n}{3} \right\rceil = \frac{n+1}{3}$.

7. (5 Points) What is a formula for the general term a_k of the sequence that starts

$$\frac{3}{5}, \frac{-5}{7}, \frac{7}{9}, \frac{-9}{11}, \frac{11}{13}, \cdots$$

8. (5 Points) Show that the square root of an irrational number is irrational.

9. (5 Points) Either prove or give a counterexample to the statement "the square of an irrational number is irrational".

10. (10 Points) Write the following using summation or product notation.
    (a) $1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3$ _____

    (b) $1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13 \cdot 15$. _____

11. (10 Points) Let $A = \{b, c, d, f, g\}$, $B = \{a, b, c\}$ and $C = \{a, f\}$ the find the following

    (a) $A - B - C$ _____

    (b) $B - (A \cup B)$ _____

12. (10 Points) Draw the Venn diagrams for the following
    (a) $A \cup (B \cap C)$

    (b) $A^c - (B \cup C)$

13. (5 points) Prove by induction that for $n \geq 0$ $1 + r + r^2 + \cdots + r^n = \dfrac{1 - r^{n+1}}{1 - r}$.