

**Mathematics 174 Test #1**

Name: \_\_\_\_\_

Show your work to get credit. An answer with no work will not get credit.

1. (5 Points) Make a truth table for  $\sim p \wedge (p \rightarrow q)$ .

| $p$ | $q$ |
|-----|-----|
|     |     |
|     |     |
|     |     |
|     |     |

2. (10 Points) Write out the negations of the following sentences:

(a) The door is open and the cat has gotten out.

(b) The weather is hot or it is humid.

(c) If the book is short, then I will read it.

(d) Every dog has fleas.

(e) Some mathematician is normal.

3. (5 Points) What is the contrapositive of the statement: "If  $x$  is even, then it will satisfy the equation."

4. (10 Points) Define the following

(a) Tautology.

(b)  $n$  is a prime number.

(c)  $b$  is a factor of  $n$ .

(d)  $m$  is an even number.

5. (10 Points) Is  $p \leftrightarrow q$  logically equivalent to  $(p \wedge q) \vee (\sim p \wedge \sim q)$ ? Justify your answer.

answer \_\_\_\_\_

Justification:

6. (5 Points) List all the elements of the set  $\{n \in \mathbb{Z} : n(n + 2) < 20\}$ .

7. (10 Points) Is the following argument valid? Justify your answer.

If Jules solved the problem correctly, then Jules obtained the answer  $x = 7$ .

Jules obtained the answer  $x = 7$ .

$\therefore$  Jules solved the problem correctly.

answer \_\_\_\_\_

Justification:

8. (15 Points) Assume that  $m$  and  $n$  are integers. Then justify your answers to the following questions.

(a) Is  $4m - 6n$  even?

answer \_\_\_\_\_

Justification:

(b) Is  $2mn^3 + 2n + 5$  odd?

answer \_\_\_\_\_

Justification:

(c) Is  $3n + 5n$  odd?

answer \_\_\_\_\_

Justification:

9. (5 Points) Show that the sum of an even integer and an odd integer is always odd.

10. (10 Points)

(a) Write  $57_{10}$  in base 2.

\_\_\_\_\_

(b) If  $x$  has binary expansion  $x = 1011001_2$  then write  $x$  as a decimal. \_\_\_\_\_

(c) Change  $4AC_{16}$  in base 16 to base 10.

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11. (5 Points) Change the repeating decimal  $12.154154154154154\dots$  to the ratio of two integers.

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12. (10 Points) Prove that if  $a$ ,  $b$ , and  $c$  are any integers and  $a \mid b$  and  $a \mid c$  then  $a \mid (2b + c)$ .

13. (Extra Credit 5 Points) If  $n$  is odd then show that  $n(n + 1)$  is even.