Math 142 Group Project # 3 Spring, 1996

This is due in class Monday April 22 (this this the last day of class). Now the standard disclaimer: You are to work in your present groups that were assigned the last time we handed out the playing cards. These groups should not be any larger than 4 persons and preferable of size 4 exactly. Each group will turn in one paper with all your names on it. As usual each name should have a percentage after it that represents the percentage of the work that the group as a whole felt that each person did. Thus if the people in the group are A, B, C, and D and everyone put about the same amount of work into the project, then everyone would be rated 25%. If however person A put in a lot of effort and person C only only did a little bit the numbers might look like A 40%, B 25%, C 10% D 25%. As long as all the numbers are above 10 % this will not effect the grade, but anyone who does less than 10 % will be penalized.

1. (The following problem comes from Prof. Miller who likes to play cards with his nephew.) The game of Old Maid is played with a deck of 33 cards consisting of 16 matching pairs and the Old Maid. The basic idea for 2 players is as follows. The dealer deals out the entire deck. Players hold their cards in hand. All players match the cards in their hands and discard all of their pairs face up. Next, the dealer draws a single card from the other player's hand. If it matches one of the cards in his hand, he discards the pair. Next, the other player draws a card from the dealer's hand. If it matches one of the cards in his hand, he discards the pair. The game continues as such. To win you must get rid of all your cards. Prof. M^2 is playing Old Maid with his nephew Ben. The game is down to the situation where Ben has one card, the Super Blob, and Prof. M^2 has two cards, which of course are the Old Maid and the other Super Blob. It is Ben's turn. The goal of this project is to intelligently answer the following question: at this point in the game, if you were to wager a bet on the game, on whom would you put your money?
   (a) What is the probability that Ben wins outright, i.e., what is the probability that Ben draws the Super Blob and not the Old Maid? Call this probability b_1.
   (b) What is the probability that Ben wins on his 2^{nd} draw, i.e., what is the probability that first Ben draws the Old Maid and then Prof. M^2 draws the Old Maid and then Ben draws the Super Blob? Call this probability b_2.
   (c) What is the probability that Ben wins on his k^{th} draw? Call this probability b_k.
   (d) What is the probability that Prof. M^2 wins on his k^{th} draw? Call this probability m_k. (Keep in mind that it is Ben's turn. So even to compute m_1 you need to consider that Ben can win before Prof. M^2 even has another chance to draw.)
   (e) Let b be the probability that Ben eventually wins and m be the probability that Prof. M^2 eventually wins. Find an "expression" (i.e., some helpful formula) for b and one for m. Then compute b and m.
   (f) Place your bet. Give an explanation of the odds that a twelve-year-old could understand.
2. The Gamma function. Read about this function on pages 685 and 686 in the text and do problems #5, #6, #7, #8, #9 and #10 on these pages. In doing #9 you are allowed to assume
$$\sqrt{2/\pi}\int_0^\infty e^{-u^2/2}\,du = 1.$$