Final

Name:____________________

Show your work! Answers that do not have a justification will receive no credit.

1. (18 Points) Find the following antiderivatives and circle your answer.

   (a) $\int (3u^4 - 9u + 7)\, du$

   (b) $\int x^2 \sin(2x)\, dx$

   (c) $\int x^2 \ln x\, dx$

   (d) $\int \frac{dx}{x(x+1)}$

   (e) $\int \tan(a\theta)\, d\theta$ (Where a is a positive constant.)

   (f) $\int e^{3v}(e^{3v} + 4)^7\, dv$

2. (20 Points) Compute the following definite integrals and circle your answer. (Some of the integrals may be divergent improper integrals. If this is the case say so.)

(a) $\int_0^5 x(5-x)\,dx$

(b) $\int_0^\infty xe^{-ax}\,dx$ (where a is a positive constant.)

(c) $\int_0^\infty \frac{dx}{1+x^2}$

(d) $\int_0^\infty \frac{x\,dx}{1+x^2}$

(e) $\int_1^4 \sqrt{2t+1}\,dt$

3. (10 Points) Solve the following initial value problems and circle your answer.

(a) $\dfrac{dy}{dx} = 2x^2 - 1, \qquad y(0) = 1.$

(b) $\dfrac{dA}{dt} = \dfrac{3t^2 + 2t}{A} \qquad A(0) = 4$

(c) $S''(t) = -32, \qquad S(0) = 64, \quad S'(0) = 4.$

4. (10 Points) A small company produces stereos. On the average one of its staff makes 4 stereos/hour. If from 8:00AM to 11:00AM two of the staff are working on making stereos and form 11:00AM to 1:00PM five of the staff are making stereos then

   (a) Graph the number of staff working as a function of time:

   (b) Graph the number of stereos made as a function of time:

   (c) Give a formula for the number of stereos made as a function of time.

5. (5 Points) Compute the average value of $f(x) = \sin(x)$ on the interval $[0, \pi]$ and circle your answer.

6. (10 Points)

   (a) State in your own words what the fundamental theorem of calculus.

   (b) Let $F(x) = \int_0^x e^{\cos(t)}\, dt$. Then what is $F'(x)$.

7. (7 Points) Sketch the graph of the solution to the initial value problem

$$y' = (4 - x^2)e^{-y^2+x^2}, \qquad y(0) = 1$$

showing where all the local maxima and minima occur.

8. (8 Points) Consider the differential equation $y' = .2y(5 - y)$.

(a) For the solution with $y(0) = 1$ what is $y'(0)$? Circle your answer.

(b) For the solution with $y(0) = 1$ what is a good estimate of $y(.1)$? Circle your answer.

(c) Graph the solution that has $y(0) = 1$.

(d) For the solution with $y(0) = 1$ what is a good estimate of $y(145.3)$. Circle your answer.

9. (10 Points) A survey shows that if a vendor at a small ball park sells hot dogs at x cents each then he will sell $300 - 2x$ of them. At what price should he sell them to maximize his profit?

10. (5 Points) Find the Taylor series for $\sin(2x)$

11. (5 Points) Find the third order Taylor series for $f(x) = \frac{1}{x}$ at the point $x = 10$.

12. (10 Points) On a cold day a stone is brought inside. By Newton's Law of cooling the stone warms so that the rate of change of its temperature is proportional to its difference in temperature with the air in the room. If the air inside is at a temperature of 65°F and the stone has a temperature of 30°F when it is first brought in then

(a) Write a rate equation satisfied by the temperature of the stone. (This equation will have a constant of proportionality coming from Newton's Law.)

(b) If after 20 minutes the stone is 40°F then what is its temperature after an hour?

13. (10 Points) Find the radius of convergence of the following series and circle you answer.

(a) $\sum\limits_{n=0}^{\infty} 5^n x^n$

(b) $\sum \dfrac{x^{2n}}{(2n+1)!}$

(c) $\sum\limits_{n=0}^{\infty} n4^n(x-5)^{2n}$

14. (6 Points) Let be a function so that $f(2) = 1$, $f'(2) = -1$, and $f''(2) = 4$.

(a) Draw a graph of $y = f(x)$ near $x = 2$.

(b) Give a good approximation to $f(1.9)$.

15. (10 Points) Compute the following and circle your answer.

(a) $\lim_{x \to 0} \dfrac{e^{2x} - 1}{3x}$

(b) $\lim_{x \to \infty} \dfrac{x^{10}}{e^x}$

(c) The sum of the series $1 + x^2 + x^4 + x^6 + x^8 + x^{10} + \cdots$

16. (6 Points) how many terms of the series

$$1 - \frac{1}{2^5} + \frac{1}{3^5} - \frac{1}{4^5} + \frac{1}{5^5} - \frac{1}{6^5} + \frac{1}{7^5} + \cdots$$

do we need to insure that we have the sum accurate to 15 decimal places. (HINT: Recall that the error at stopping at the n-th term of an alternating series is less than the size of the next term in the series.)

Have a good summer!