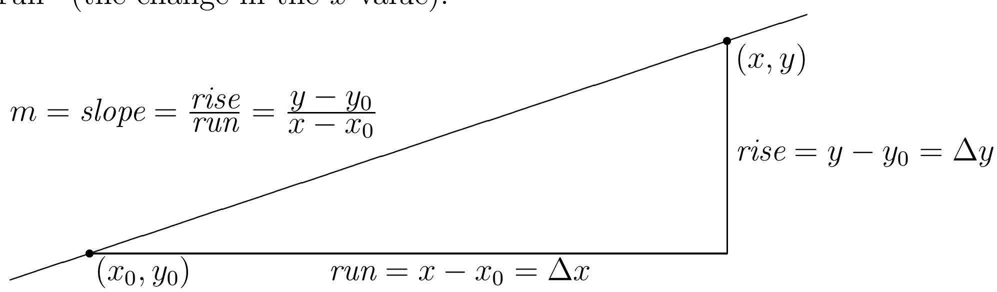
Homework Due Thursday August 29

Problems on Linear and Piecewise Linear Functions

Two variables, say x and y, are said to be ***linearly related*** if and only if there are constants m and b so that $y = mx + b$. Thus if x and y are related by the equation $4x - 2y = 8$ then they are linearly related as it is possible to solve for y to get $y = 2x - 4$ (in this case $m = 2$ and $b = -4$). Likewise if the variables A and R are related by the equation $5A - 3R = 7$ then A and R are linearly related as it is possible to solve for A and get $A = \frac{5}{3}R + \frac{7}{3}$ (whence this time $m = \frac{5}{3}$ and $b = \frac{7}{3}$).

The basic most basic fact about a linear relation is that its graph is a straight line. As we all know the ***slope*** of this line is the "rise" (i.e. the change in the y value) divided by the "run" (the change in the x value).



The slope m is the same regardless of which pair of points (x, y) and (x_0, y_0) are used. Thus if we think of (x_0, y_0) as a fixed point and (x, y) as a variable point then the change in x is $\Delta x = x - x_0$ and the change in y is $\Delta y = y - y_0$. Then Δx and Δy are also variables and the equation of the line can be written as

$$\Delta y = m\Delta x \quad \text{where} \quad \Delta x = x - x_0,\ \Delta y = y - y_0.$$

This is the same as $y - y_0 = m(x - x_0)$ which is what is often called the point slope form of the line.

Here are some problems to review what you know about linear functions and to introduce you to piecewise linear functions.

1. If a car is moving at constant velocity, then the relation between distance, D, and time, t, is linear. For the rest of this problem it will be assumed that a car is moving a constant velocity along a straight road running by campus, that after one hour it is 10 miles from campus and after three hours it is 50 miles from campus.

   (a) What is the speed of the car in miles per hour?

   (b) Give a formula for the distance D. Let $\Delta t = t - 1$ and $\Delta D = D - 10$ then write this equation in the form $\Delta D = m\Delta t$.

   (c) Using your answer to last part graph D as a function of t.

   (d) Use your equation to predict when the distance of the car from campus when $t = 5$ hours.

(e) What is the time when the car is at a distance of 170 miles from campus?

2. In this problem we assume assume, just as in the last problem, that a car is moving at constant velocity.

   (a) If the velocity is 14mph, then how far does the car travel in 30 minutes? How far does in travel in 1.5 hours? What is a general formula for the change ΔD in D in terms of a change Δt in time?

   (b) If at time $t = 1.5$ hours the distance from campus is 45 miles, and the velocity is 50mph, then predict the distance of the car from campus when $t = 4$ hours. Also graph D as a function of t.

3. This problem is very much like the jogger problem we did earlier. The only real difference (other than working with cars rather than runners) is this time we want to get some formulas and not just the graph. Assume that the at time $t = 0$ our car from the last two problems car starts from campus and for the first hour it moves at a constant velocity of 30mph. After this hour is up it gets on the interstate and goes at a constant velocity of 60mph for 2 hours. Then it starts to rain and the driver slows to 45mph for the next hour. As before let D be the distance from campus t the time measured in hours.

   (a) Find a formula for D in terms of t in the interval $0 \le t \le 1$.

   (b) How far has the car traveled after the first hour.

   (c) Find a formula for D in terms of t in the interval $1 \le t \le 3$. (This is the interval where the car is traveling at 60mph.)

   (d) How far has the car traveled when $t = 3$?

   (e) Find a formula for D in terms of t in the interval $3 \le t \le 4$. (This is the interval where the car is going at 45mph.)

   (f) Graph the velocity as function of time. This will be a "piecewise constant" in the sense that on each of the intervals $(0, 1)$, $(1, 3)$ and $(3, 4)$ the function is constant.

   (g) Graph D as a function of t on the interval $0 \le t \le 4$. The result should be "piecewise linear" in the sense that it is three straight line segments joined together.

Problems on proportions

In mathematics the word ***proportional*** means "is a constant multiple of". That is $f(x)$ and $g(x)$ are proportional iff there is a constant C so that $f(x) = Cg(x)$. The constant C is called the ***constant of proportionality***. For example the area of a circle is proportional to the square of its radius and the constant of proportionality is π. That is if A is the area and r is the radius, so that the square of the radius is r^2, then $A = \pi r^2$. Likewise the area A of a triangle is proportional to the product of its base b height h. That is $A = Cbh$. And we know from high school geometry that the constant of proportionality is $C = \frac{1}{2}$. As practice in using this language answer the following:

1. The cost C of a box of chocolates is directly propositional to its weight W. Write a formula relating C and W. (Note that in this problem you should call the constant of proportionality something other than C.)
   Answer: __________________

2. The energy E of a bullet is proportional to product of its mass m the square of its speed v.
   Answer:__________________

3. The rate of change T' of the temperature T is proportional to the difference of T with the temperature of the air which is measured to be 70°F.
   Answer:__________________

4. The cost C of a pizza is proportional to the square of its diameter D.
   Answer:__________________

5. If y is proportional to x then draw the graph of y as a function of x.

6. If V is proportional to the cube of r (that is if V is proportional to r^3) then draw the graph of V as a function of r.