Test 3 Name:____________________________

Show your work! Answers that do not have a justification will receive no credit.

1. (25 points) Find the indicated derivatives.

(a) $w(t) = -3t^6 + \sqrt{t^3+1}$.

$w'(t) =$

(b) $f(\theta) = e^{2\theta}(\cos\theta + 2\tan\theta)$

$f'(\theta) =$

(c) $z = \dfrac{x+y}{x-y}$

$\dfrac{\partial z}{\partial x} =$

$\dfrac{\partial^2 z}{\partial x \partial y} =$

(d) $S(t) = t^2 e^{2t+1}$

$S''(t) =$

2. (10 points) Find a and k so that $y = ae^{kt}$ is a solution to the initial value problem

$$y' = -3.7y, \quad y(0) = 13.5$$

3. (20 points) (a) The cost of a college education is increasing at an increasing rate. Graph cost as a function of time.

(b) If $y'(x) = \dfrac{4 - x^2}{1 + y^4}$ and $y(0) = 0$ make a rough sketch of the graph of $y(x)$ on the interval $-3 \leq x \leq 3$ and **label the values of x where all local maxima and minima occur**.

(c) Let $H(x)$ be a function with $H(-1) = 2$, $H'(-1) = \frac{1}{2}$ and $H''(-1) = -3$. Make a rough graph of $y = H(x)$ near $x = -1$.

(d) The following is the graph of $y = f(x)$. On the same axis make sketch the graph of the derivative.

4. (10 points) (a) Write the full microscope equation for the function $f(x, y) = 2x^3 - y^2x$ at the point where $x = 1$, and $y = 2$.

(b) If f is increased by .5 and y is decreased by .2 then approximate the change in x.

5. (15 points) Let $H(s)$ be a solution to the initial value problem

$$H'(s) = -2(H(s)+2)(H(s)-3), \qquad H(2) = 1.$$

(a) Make a rough sketch of the graph of $H(s)$ on the interval $-4 \leq s \leq 4$.

(b) Approximate $H(532.7)$

(c) Approximate $H(1.8)$.

6. (10 points) Find the smallest value of $y = x^2 + \frac{16}{x}$ on the interval $(0, \infty)$. Where does this smallest value occur?

7. (10 points) When a cold spoon is used to stir a hot cup of coffee, the coffee cools down and the spoon heats up. By Newton's law of cooling the rate of changes of the temperatures of both the coffee and the spoon are proportional to the difference in their temperatures. Write a system of differential equations for the temperatures of the coffee and the spoon labeling all independent and dependent variables.